ENGINEERING MATH-I

Semester: 1^{ST}

STUDY MATERIAL

ENGINEERING MATH-I

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MATRICES

→ A matrix is a rectangular array in which elements are arranged in terms of rows and columns.

→ The horizontal lines are called rows. The vertical lines are called columns.

Let A matrix consist of m no. of rows and n no. of columns, the order of matrix A is m×n.

Ex:- let, $A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & -2 \end{bmatrix}_{3\times 2}$

Diogonal of a matrix:-

let a_{ij} denotes elements in a matrix A.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2m} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mm} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 5 & 3 \\ 3 & 7 & 2 \end{bmatrix}_{3\times 3} \qquad \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}_{2\times 2}.$$

Trace = ~~Some~~ Sum of the diogonal elements

$$\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} = 3.$$

$$\begin{bmatrix} 3 & 2 & 5 & 7 \\ 0 & -9 & 2 & 3 \\ 0 & -1 & -2 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = 3-9-2+1 = -7.$$

Types of matrices :-

(i) Square matrix :- no. of rows = no. of columns

Ex :- $[2]_{1\times1}$ $[-4]_{1\times1}$ $\begin{bmatrix} 2 & 0 \\ 5 & 2 \end{bmatrix}_{2\times2}$ $\begin{bmatrix} -3 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}_{3\times3}$

(ii) Diagonal matrix :- If in a matrix the diagonal elements are nonzero and other elements are zero then it is said to be a diagonal matrix.

Ex :- $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$, $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -7 \end{bmatrix}$.

(iii) Scalar matrix :- If in a matrix the diagonal elements are non zero and equal but other elements are zero, then it is said to be a scalar matrix.

Ex :- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$.

(iv) Identity or unit matrix :-

It is denoted by I.

If in a matrix the diagonal elements are 1 and other elements are zero then it is said to be an identity or unit matrix.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$.

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$.

Addition and subtraction of two matrices:

A+B }
A-B } A and B has the same order

$$\begin{bmatrix} 3 & 0 & -2 \\ 5 & 3 & 4 \end{bmatrix}_{2\times3} + \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}_{2\times3} = \begin{bmatrix} 4 & 0 & -1 \\ 7 & 6 & 8 \end{bmatrix}_{2\times3}$$

$$k\begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3k & -4k \\ 2k & 5k \end{bmatrix}$$

- where $\boxed{k = \text{constant}}$.

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Transpose of a matrix:

Let A be a matrix of order m×n.
The Transpose of A is denoted by A^T and has the order n×m.

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Multiplication of two matrices:

Ex:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3} \begin{bmatrix} 0 & -1 \\ 1 & 2 \\ 0 & 0 \end{bmatrix}_{3\times2}.$$

$$= \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}_{2\times2}.$$

Let A be a matrix of order m×n.
and B be a matrix of order n×p.
The product of A and B can be written as m×p
$A \times B = C$.

Soln of system of eqn by cramer's rule :

let $a_1x + b_1y = c_1$ —(1) and
$a_2x + b_2y = c_2$ —(2).

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$\Delta x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$$

$$\Delta y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - a_2c_1 .$$

$$x = \frac{\Delta x}{\Delta} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{\Delta y}{\Delta} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}.$$

let $a_1x + b_1y + c_1z = d_1$ —(1)
$a_2x + b_2y + c_2z = d_2$ —(2)
$a_3x + b_2y + c_2z = d_3$ —(3)

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.$$

$$\Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad \Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}.$$

$$\Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$$

$$x = \frac{\Delta x}{\Delta}, \quad y = \frac{\Delta y}{\Delta}, \quad z = \frac{\Delta z}{\Delta}.$$

(Q) solve by cramer's Rule

$$2x - y = 2 \quad -(1)$$
$$3x + y = 13 \quad -(2).$$

$$\Delta = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2+3 = 5$$

$$\Delta x = \begin{vmatrix} 2 & -1 \\ 13 & 1 \end{vmatrix} = 2+13 = 15.$$

$$\Delta y = \begin{vmatrix} 2 & 2 \\ 3 & 13 \end{vmatrix} = 26 - 6 = 20$$

$$x = \frac{\Delta x}{\Delta} = \frac{15}{5} = 3.$$

$$y = \frac{\Delta y}{\Delta} = \frac{20}{5} = 4.$$

Adjoint of a matrix :-

minor — M_{ij}

cofactor — C_{ij}

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$$

minor	cofactor
$M_{11} = -3$	$C_{11} = -3$
$M_{12} = 2$	$C_{12} = -2$
$M_{21} = -1$.	$C_{21} = +1$
$M_{22} = 1$.	$C_{22} = 1$.

$$C_{ij} = \begin{bmatrix} -3 & -2 \\ 1 & 1 \end{bmatrix}.$$

$$\therefore \text{adj}A = [C_{ij}]^T.$$

$$= \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}.$$

Inverse of a matrix:

Let A be a matrix of order $n \times n$. The inverse of A is denoted by A^{-1}.

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} \quad \text{where, } |A| \neq 0.$$

Q:- find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$|A| = 10 + 12 = 22 \neq 0.$$

Minor	Cofactor
$M_{11} = 5$	$C_{11} = 5$
$M_{12} = 4$	$C_{12} = -4$
$M_{21} = -3$	$C_{21} = 3$
$M_{22} = 2$	$C_{22} = 2.$

$$C_{ij} = \begin{bmatrix} 5 & 3 \\ -4 & 2 \end{bmatrix}.$$

$$\text{Adj } A = \begin{bmatrix} 5 & 3 \\ -4 & 2 \end{bmatrix}.$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{22}\begin{bmatrix} 5 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ \frac{-4}{22} & \frac{1}{11} \end{bmatrix}.$$

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soln of system of eqn by Matrix inverse method :

$2x+3y=6$

$x+2y=4$.

The co-efficient matrix is given by

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 4 \end{bmatrix}.$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1.$$

Minor

$M_{11} = 2$

$M_{12} = 1$

$M_{21} = 3$

$M_{22} = 2$

cofactor

$C_{11} = 2$

$C_{12} = -1$

$C_{21} = -3$

$C_{22} = 2$.

$$\text{adj } A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

The above system of eqn can be written as,

$A \cdot X = B$

$\Rightarrow X = A^{-1}B$.

$$A^{-1} = \frac{1}{|A|} \times \text{adj } A.$$

$$= \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

$$\therefore X = A^{-1}B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 12 \\ -6 + 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \Rightarrow x = 0 \text{ and } y = 2.$$

Dt-23.11.21

(9) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$.

$$A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

L.H.S // $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \text{ R.H.S} \quad \text{(proved)}.$$

1) Find X and Y if

$$X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \text{ (1)} \quad \text{and} \quad X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \text{ (2)}.$$

Add eqn (1) & (2)

$$X+Y+X-Y = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}.$$

Put the value of X in eqn (1)

$$\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow Y \ldots$$

(Q) Find x and y if $2x+3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ —(1)

$$3x+2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \text{ —(2)}$$

$$eq^n\ (1) \times 3 = 6x + 9y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} \text{ —(3)}$$

$$eq^n\ (2) \times 2 = 6x + 4y = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix} \text{ —(4)}$$

~~adding~~ substrate eq^n (3) & (4)

$$\cancel{6x + 9y - 6x - 4y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}}$$

$$\Rightarrow 5y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}.$$

$$\cancel{6x} + 9y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$$

$$\cancel{6x} + 4y = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

(−) (−) (−)

$$5y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

$$\Rightarrow y = \frac{1}{5}\begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}.$$

Put the value of Y in eq^n(1)

$$2x + \frac{3}{5}\begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2x + \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & \frac{-30}{5} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 6/5 & 39/5 \\ 42/5 & -30/5 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{2}\begin{bmatrix} 4/5 & -24/5 \\ \frac{-22}{5} & 30/5 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 4/10 & -24/10 \\ \frac{-22}{10} & 30/10 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

(8) $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

$$2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

9) $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$

$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$

$\Rightarrow 2\cancel{x}+y = 5$ and $2x+2 = 8.$

$\Rightarrow y = 5-2 = 3$ $\Rightarrow 2x = 6$

$\Rightarrow x = 3.$

10) $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}.$

$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}.$

$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} = \begin{bmatrix} 6 & 18 \\ 12 & 12 \end{bmatrix}$

$\Rightarrow 2x = 6, \quad 2z = 18, \quad 2y = 12, \quad 2t = 12$

$\Rightarrow \boxed{x = 3}, \Rightarrow \boxed{z = 9}, \Rightarrow \boxed{y = 6}, \Rightarrow \boxed{t = 6}$

(11) $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow 2x - y = 10$ —(1)

$\Rightarrow 3x + y = 5$ —(2)

Add eq^n (1) & (2)

$$2x - y + 3x + y = 15$$

$$\Rightarrow 5x = 15$$

$$\Rightarrow x = \cancel{15}\ \cancel{5}\ 3.$$

Put the value of x in ~~eqn (1)~~ eq^n(1)

$$2x - y = 10$$

$$\Rightarrow 6 - y = 10$$

$$\Rightarrow -y = 4$$

$$\Rightarrow y = -4.$$

(12) $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}.$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} 4+x & 6+x+y \\ z+w-1 & 3+2w \end{bmatrix}$$

$\Rightarrow 3x = 4+x$, $3y = 6+x+y$, $3z = z+w-1$, $3w = 3+$

$\Rightarrow 2x = 4$, $\Rightarrow 2y = 8$, $\Rightarrow 2z = 3-1$, $\Rightarrow w = 3.$

$\Rightarrow x = 2.$ $\Rightarrow y = 4.$ $\Rightarrow 2z = 2$

$\Rightarrow z = 1.$

Dt. 1-12-21

Determinant :

Properties of determinant :

(1) ~~Do~~ $|A| = |A^T|$.

(2) If all the elements of ~~dete~~ any row and or column of a determinant are zero then the determinant is zero.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0.$$

(3) In a matrix if any two ~~de~~ rows or columns are identical then determinant equal to zero.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 5 \\ 1 & 2 & 3 \end{bmatrix} = 0.$$

(4) If we interchange two adjacent rows or adjacent columns of a determinant then the absolute value of the determinant remain same but changes the sign.

$R_1 \leftrightarrow R_2$.

(5) $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$\Delta = a_{11} c_{11} + a_{12} c_{12} + a_{13} c_{13}$.

(Q) Let A be a matrix of order 3x3 find determinant of $|3A| = 27|A|$.

$$3\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 3a_{11} & 3a_{12} & 3a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3a_{31} & 3a_{32} & 3a_{33} \end{bmatrix}.$$

$$= 3 \cdot 3 \cdot 3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= 27 \, |A| .$$

(Q) find the determinant of $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.

Ans:-

$\cos^2\theta + \sin^2\theta = 1.$
$\sec^2\theta - \tan^2\theta = 1$
$\mathrm{cosec}^2\theta - \cot^2\theta = 1$.

$\mathrm{cosec}\,\theta = \dfrac{1}{\sin\theta}$.
$\sec\theta = \dfrac{1}{\cos\theta}$
$\cot\theta = \dfrac{1}{\tan\theta}$.

Singular matrix :-

If the determinant of a matrix is equal to zero then it is said to be a ~~sig~~ singular matrix.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0.$$

(Q) - $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0.$

Since first two rows or two columns are identic

(Q) If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$. find x.

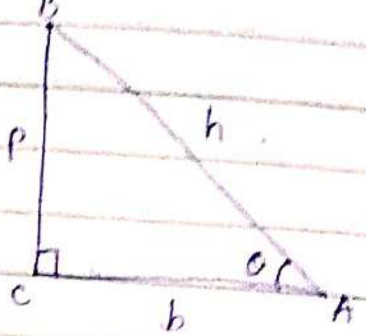
Ans:- $x = a$.

6.12.21

Trigonometry :

In trigonometry we have 6 trigonometric func i.e $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, $\operatorname{cosec}\theta$.

Let A, B, C be a right angle triangle where $\angle A = \theta$.



$\sin\theta = \dfrac{\text{opposite side of } \theta}{\text{hypotenous}}$

$= \dfrac{p}{h}$

$\cos\theta = \dfrac{\text{Adjacent side of } \theta}{\text{hypotenous}} = \dfrac{b}{h}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{p}{h} \times \dfrac{h}{b} = \dfrac{p}{b}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{b}{h} \times \dfrac{h}{p} = \dfrac{b}{p}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{h}{b}$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{h}{p}$

$\dfrac{1}{\operatorname{cosec}\theta} = \sin\theta$, $\dfrac{1}{\sec} = \cos\theta$, $\dfrac{1}{\tan\theta} = \cot\theta$

(1) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

(2) $\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

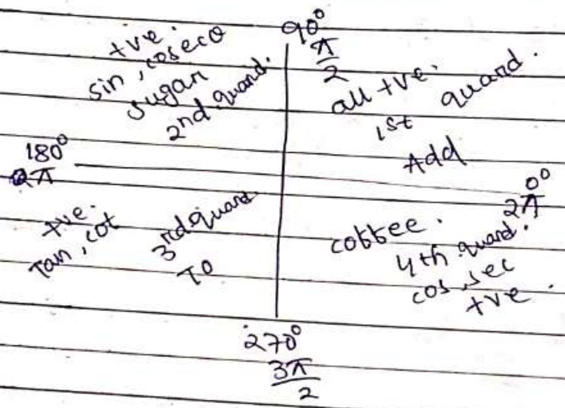
$\tan^2\theta = \sec^2\theta - 1$

(3) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1



compound angle :-

$$\boxed{\begin{aligned}
\sin(A+B) &= \sin A \cdot \cos B + \cos A \cdot \sin B. \\
\sin(A-B) &= \sin A \cdot \cos B - \cos A \cdot \sin B. \\
\cos(A+B) &= \cos A \cdot \cos B - \sin A \cdot \sin B. \\
\cos(A-B) &= \cos A \cdot \cos B + \sin A \cdot \sin B. \\
\sin 2A &= 2\sin A \cdot \cos A. \\
\cos 2A &= \cos^2 A - \sin^2 A. \\
\cos 2A &= 1 - 2\sin^2 A.
\end{aligned}}$$

$2\sin^2 A = 1 - \cos^2 A$.

$\sin^2 A =$

$\cos 2A = \cos^2 A - (1 - \cos^2 A)$

$\Rightarrow \cos 2A = \cos^2 A - 1 + \cos^2 A$

$\Rightarrow \cos 2A = 2\cos^2 A - 1$.

$\Rightarrow \boxed{2\cos^2 A = 1 + \cos 2A}$.

$$\sin C + \sin D = 2\sin\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2} \cdot \sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2} \cdot \sin\frac{C-D}{2}$$

Dt. 7.12.21

$\sin(-\theta) = -\sin\theta$.
$\cos(-\theta) = \cos\theta$.
$\tan(-\theta) = -\tan\theta$.
$\cot(-\theta) = -\cot\theta$.
$\sec(-\theta) = \sec\theta$.
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$.

$\sin(\frac{\pi}{2} + \theta) = \cos\theta$
$\cos(\frac{\pi}{2} + \theta) = -\sin\theta$.
$\tan(\frac{\pi}{2} + \theta) = -\cot\theta$.
$\cot(\frac{\pi}{2} + \theta) = -\tan\theta$.
$\sec(\frac{\pi}{2} + \theta) = -\operatorname{cosec}\theta$.
$\operatorname{cosec}(\frac{\pi}{2} + \theta) = \sec\theta$.

$\sin(\frac{\pi}{2} - \theta) = \cos\theta$.
$\cos(\pi/2 - \theta) = \sin\theta$.
$\tan(\pi/2 - \theta) = \cot\theta$.
$\cot(\pi/2 - \theta) = \tan\theta$.
$\sec(\pi/2 - \theta) = \operatorname{cosec}\theta$.
$\operatorname{cosec}(\pi/2 - \theta) = \sec\theta$.

$\sin(\pi - \theta) = \sin\theta$
$\cos(\pi - \theta) = -\cos\theta$.
$\tan(\pi - \theta) = -\tan\theta$.
$\cot(\pi - \theta) = -\cot\theta$.
$\sec(\pi - \theta) = -\sec\theta$
$\operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta$

$$\sin(\pi+\theta) = -\sin\theta$$
$$\cos(\pi+\theta) = -\cos\theta$$
$$\tan(\pi+\theta) = \tan\theta$$
$$\cot(\pi+\theta) = \cot\theta$$
$$\sec(\pi+\theta) = -\sec\theta$$
$$\operatorname{cosec}(\pi+\theta) = -\operatorname{cosec}\theta$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \cdot \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \cdot \tan B}$$

$$\sin(2\pi+\theta) = \sin\theta$$
$$\cos(2\pi+\theta) = \cos\theta$$
$$\tan(2\pi+\theta) = \tan\theta$$
$$\cot(2\pi+\theta) = \cot\theta$$
$$\sec(2\pi+\theta) = \sec\theta$$
$$\operatorname{cosec}(2\pi+\theta) = \operatorname{cosec}\theta$$

$$\cot(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$$

$$\cot(A-B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$$

Q:- Find $\sin(210)^\circ = \sin(180^\circ + 30^\circ)$
$= -\sin 30^\circ$
$= -\frac{1}{2}$.

Find $\cos(315)^\circ = \cos(180^\circ + 135^\circ)$
$= -\cos 135^\circ$
$= -\cos(180^\circ - 45^\circ)$
$= \cos 45^\circ = \frac{1}{\sqrt{2}}$.

(Q) Evaluate $\sin(1185^\circ)$
~~$= \sin(360^\circ +$~~
$= \sin(3 \times 360^\circ + 105^\circ)$

$= \sin 105^\circ$
$= \sin(90^\circ + 15^\circ)$.
$= \cos 15^\circ$.

(9.) $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n=\begin{cases}2\cot^n\left(\frac{A-B}{2}\right), & n \text{ is even} \\ 0 & n \text{ is odd}\end{cases}$

L.H.S. $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$.

$=\left(\frac{\cancel{2}\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}}{\cancel{2}\cos\frac{A+B}{2}\cdot\sin\frac{A-B}{2}}\right)^n+\left(\frac{2\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}}{-2\sin\frac{A+B}{2}\cdot\sin\frac{A-B}{2}}\right)^n$.

$=\left(\cot\frac{A-B}{2}\right)^n+\left(-\cot\frac{A-B}{2}\right)^n$.

$=\cot^n\frac{A-B}{2}+(-1)^n\cot^n\frac{A-B}{2}$.

If n is even, $\cot^n\left(\frac{A-B}{2}\right)+\cot^n\left(\frac{A-B}{2}\right)$

$=2\cot^n\frac{A-B}{2}$

If n is odd, $\cot^n\left(\frac{A-B}{2}\right)-\cot^n\left(\frac{A-B}{2}\right)$

$=0.$ (Proved).

9) If $A+B+C=\pi$ then prove that $\sin 2A+\sin 2B+\sin 2C=4\sin A\cdot\sin B\cdot\sin C$.

Ans:- $\sin 2A+\sin 2B+\sin 2C$.

$=2\sin\frac{2A+2B}{2}\cdot\cos\frac{2A-2B}{2}+2\sin C\cdot\cos C$.

$$= 2\sin\frac{2(A+B)}{2}\cdot\cos\frac{2(A-B)}{2} + 2\sin C\cdot\cos C$$

$$= 2\sin(A+B)\cdot\cos(A-B) + 2\sin C\cdot\cos C.$$

$$= 2\sin(\pi-C)\cdot\cos(A-B) + 2\sin C\cdot\cos C.$$

$$= 2\sin C\cdot\cos(A-B) + 2\sin C\cdot\cos C.$$

$$= 2\sin C\,(\cos(A-B) + \cos C).$$

$$= 2\sin C\,\{\cos(A-B) + \cos\{\pi-(A+B)\}\}$$

$$= 2\sin C\,\{\cos(A-B) - \cos(A+B)\}.$$

$$= 2\sin C\left(-2\sin\frac{A-B+A+B}{2}\cdot\sin\frac{A-B-A-B}{2}\right)$$

$$= 2\sin C\,(-2\sin A\cdot\sin(-B))$$

$$= 2\sin C\,(2\sin A\cdot\sin B).$$

$$= 4\sin A\cdot\sin B\cdot\sin C \quad \text{(Proved)}.$$

Half angle formulas:-

$$\sin\theta = 2\cos\frac{\theta}{2}\cdot\cos\frac{\theta}{2}.$$

$$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

$$2\cos^2\frac{\theta}{2} = 1+\cos\theta.$$

$$2\sin^2\frac{\theta}{2} = 1-\cos\theta.$$

$$\sin 2\theta = 2\sin\theta\cdot\cos\theta$$

$$= \frac{2\tan\theta}{1+\tan^2\theta}.$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

Inverse trigonometric funⁿ :

Dt. 14.12.2[illegible]

$\sin^{-1}(\sin x) = x$.
$\cos^{-1}(\cos x) = x$.
$\tan^{-1}(\tan x) = x$.
$\cot^{-1}(\cot x) = x$.
$\sec^{-1}(\sec x) = x$.
$\mathrm{cosec}^{-1}(\mathrm{cosec}\, x) = x$.

Range of Inverse Funⁿ :-

$\sin^{-1}x = -\pi/2 \le x \le \pi/2$.

$\cos^{-1}x = 0 \le x \le \pi$.

$\tan^{-1}x = -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

$\cot^{-1}x = 0 \le x \le \pi$.

$\sec^{-1}x = 0 \le x \le \pi$.

$\mathrm{cosec}^{-1}x = -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

Note-1 :-

$\sin^{-1}\frac{1}{x} = \mathrm{cosec}^{-1}x$.

$\tan^{-1}\frac{1}{x} = \cot^{-1}x$.

$\cos^{-1}\frac{1}{x} = \sec^{-1}x$.

Note-2 :-

$\sin^{-1}x + \cos^{-1}x = \pi/2$.
$\sec^{-1}x + \mathrm{cosec}^{-1}x = \pi/2$.
$\tan^{-1}x + \cot^{-1}x = \pi/2$.

Note-3 :-

$$\sin^{-1}(-x) = -\sin^{-1}x.$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}x.$$

$$\tan^{-1}(-x) = -\tan^{-1}x.$$

$$\cot^{-1}(-x) = -\cot^{-1}x.$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}x.$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x.$$

Note-4 :-

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right).$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right).$$

Note-5 :-

$$\sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right).$$

$$\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right).$$

$$\cancel{\tan^{-1}x = \sin^{-1}\sqrt{\frac{x}{1+x^2}}}$$

$$\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right).$$

Note-6 :-

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

$$\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{y} + \tan^{-1}\frac{1}{z}.$$

Note-7 :-

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$
$$= \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$
$$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Note-8 :-

$$\sin n\pi = 0.$$
$$\cos n\pi = (-1)^n$$

$$2\sin^{-1}x = \sin^{-1} 2x\sqrt{1-x^2}$$

$$2\cos^{-1}x = \cos^{-1}(2x^2-1)$$
$$3\sin^{-1}x = \sin^{-1}(3x-4x^3),$$
$$3\cos^{-1}x = \cos^{-1}(4x^3-3x)$$

$$3\tan^{-1}x = \tan^{-1}(2x\sqrt{1-x^2})$$

Note-9 :-

$$\sin^{-1}x+\sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right].$$
$$\sin^{-1}x-\sin^{-1}y = \sin^{-1}x\left[x\sqrt{1-y^2}\cancel{\phi}-y\sqrt{1-x^2}\right].$$

$$\cos^{-1}x+\cos^{-1}y = \cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right).$$

$$\cos^{-1}x-\cos^{-1}y = \cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right).$$

Dt. 16.12.21

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ then prove that,

$x^2+y^2+z^2+2xyz=1$.

Given that, $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$.

$\Rightarrow \cos^{-1}x+\cos^{-1}y=\pi-\cos^{-1}z$.

$\Rightarrow \cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)=\pi-\cos^{-1}z$

$\Rightarrow \cancel{\cos^{-1}}\ xy-\sqrt{1-x^2}\sqrt{1-y^2})=\cos(\pi-\cos^{-1}z)$.

$\Rightarrow xy-\sqrt{1-x^2}\sqrt{1-y^2}=-\cos(\cos^{-1}z)$.

$\Rightarrow xy-\sqrt{1-x^2}\sqrt{1-y^2}=-z$

$\Rightarrow xy+z=\sqrt{1-x^2}\sqrt{1-y^2}$.

Taking square on both sides,

$\Rightarrow (xy+z)^2=(\sqrt{1-x^2}\sqrt{1-y^2})^2$.

$\Rightarrow x^2y^2+z^2+2xyz=(1-x^2)(1-y^2)$.

$\Rightarrow x^2y^2 + z^2 + 2xyz = 1 - y^2 - x^2 + x^2y^2$

$\Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$. (Proved)

2) Evaluate $2\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{4}$.

$$= \tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right) - \tan^{-1}\frac{1}{4}$$

$$= \tan^{-1}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right) - \tan^{-1}\frac{1}{4}$$

$$= \tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right) - \tan^{-1}\frac{1}{4}$$

$$= \tan^{-1}\left(\frac{\cancel{2}}{\cancel{5}} \times \frac{\cancel{25}^{\,5}}{\cancel{24}_{\,12}}\right) - \tan^{-1}\frac{1}{4}$$

$$= \tan^{-1}\frac{5}{12} - \tan^{-1}\frac{1}{4}$$

$$= \tan^{-1}\left(\frac{\frac{5}{12} - \frac{1}{4}}{1 + \frac{5}{48}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5-3}{12}}{\frac{53}{48}}\right)$$

$$= \tan^{-1}\left(\frac{2}{\cancel{12}} \times \frac{\cancel{48}^{\,4}}{53}\right) = \tan^{-1}\left(\frac{8}{53}\right).$$ (Ans)

~~Sin~~ Find the max^m and min^m value of
$5\sin x + 12\cos x$.

put $5 = r\cos\theta$. $\Rightarrow r^2\cos^2\theta = 25$
$12 = r\sin\theta$. $\Rightarrow r^2\sin^2\theta = 144$.

Now $r^2\cos^2\theta + r^2\sin^2\theta = 169$.

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 169$.

$\Rightarrow r = 13$.

$\boxed{\sin \to [-1, 1]}$

Now, $13\cos\theta \cdot \sin x + 13\sin\theta \cdot \cos x$.

$= 13\sin(x+\theta)$.

maximum value $= 13 \times 1 = 13$.
minimum value $= 13 \times (-1) = -13$.

9/ $8\cos x - 15\sin x - 2$.

Put $8 = r\sin\theta \Rightarrow r^2\sin^2\theta = 64$.
$15 = r\cos\theta$. $\Rightarrow r^2\cos^2\theta = 225$.

Now,

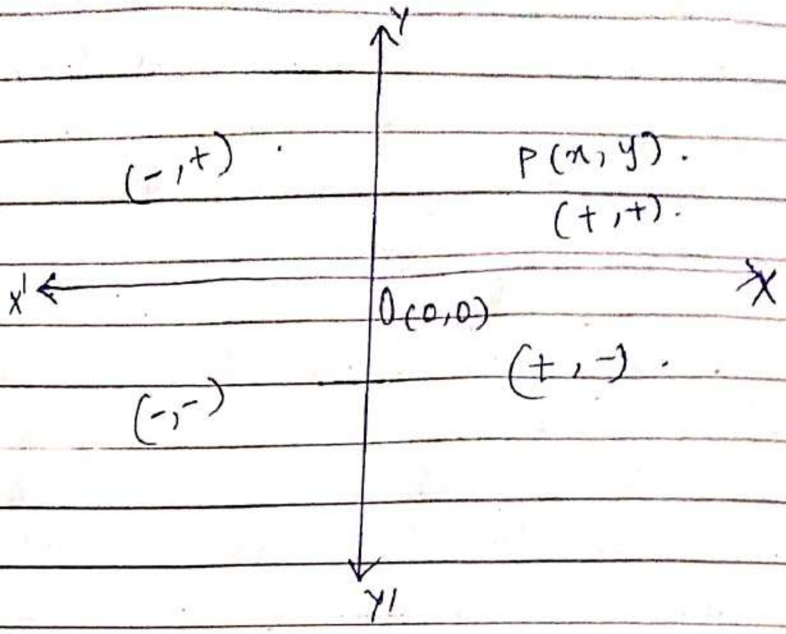
$r^2\sin^2\theta + r^2\cos^2\theta = 289$
$\Rightarrow r^2(\sin^2\theta + \cos^2\theta) = 289$
$\Rightarrow r = 17$.

Now, $17\sin\theta \cdot \cos x - 17\cos\theta \cdot \sin x$.
$= 17\sin(\theta - x) - 2$.
max^m value $= 17 \times 1 - 2 = 15$
min^m value $= 17 \times (-1) - 2 = -17 - 2 = -19$.

Introduction to co-ordinate Geometry in 2-Dimension

Let

$x'x$ and yy' are two mutually perpendicular lines intersecting at the point 'o'.



Distance formula :-

Let P and Q are the two points having coordinates (x_1, y_1) and (x_2, y_2) respectively

P (x_1, y_1) ——— Q (x_2, y_2).

$$\boxed{|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

Division formula or section formula :-

The co-ordinates of a point $C(x, y)$ divides the line Joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m:n$ is given by

$$\boxed{x = \frac{mx_2 + nx_1}{m+n} \qquad y = \frac{my_2 + ny_1}{m+n}}$$

If it divides externally,

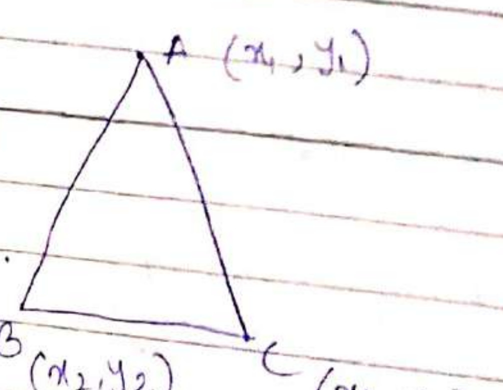
$$\boxed{x = \frac{mx_2 - nx_1}{m-n}, \qquad y = \frac{my_2 - ny_1}{m-n}}$$

mid-Point formula:-

$$\boxed{x = \frac{x_1 + x_2}{2}, \qquad y = \frac{y_1 + y_2}{2}}.$$

Area of a triangle:-

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.



or

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Dt. 17.12.21

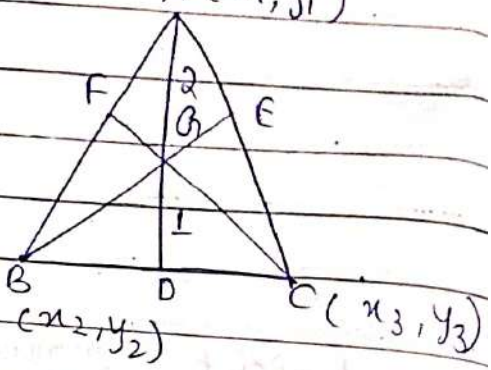
Co-linearity of three points :-

Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are said to be colinear if $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$.

$\Rightarrow$ Area of $\Delta ABC = 0$.

Centroid of a triangle :-

The point at which medians of a triangle intersect is called the centroid of the triangle.

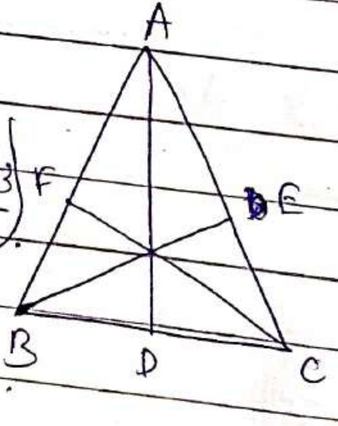


ratio = 2 : 1.

$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

Incentre :-

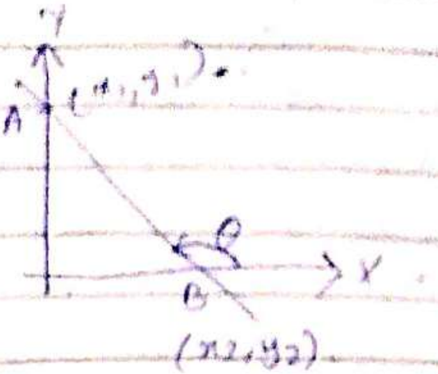
The point at which the bisectors of the angles of a triangle intersect is called the incentre of the triangle.



$$I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_3+cy_3}{a+b+c}\right).$$

where $a = BC$, $b = AC$, $c = AB$.

Slope of a line :



Let a line makes the angle θ with the +ve dirn of x-axis. This is known as angle inclination

If θ is the inclination of a line then the value of $\tan\theta$ is called slope of the line and it's denoted by

$$\boxed{m = \tan\theta \quad \text{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Note-1 :-

The slope of a line parallel to x-axis is zero.

$\Rightarrow$ slope of x-axis $= 0$.

Note-2 :-

The slope of y-axis is not defined because $\theta = 90°$.

$\Rightarrow M = \tan 90° = \infty$.

Note-3 :-

Two lines are parallel if and only if their slopes are equal.

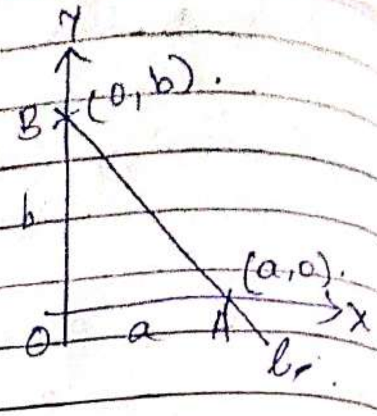
$\Rightarrow m_1 = m_2$.

Note-4 :-

Two lines are said to be perpendicular. Iff

If $m_1 m_2 = -1$.

Intercepts:-



Let AB is a line intercepting x and y axis at the point A, B respectively.
Then x intercept = OA = a.
Y intercept = OB = b.

eqn of straight lines:-

(1) Slope - Intercept form.

⇒ $\boxed{y = mx + c}$ where c = y-intercept.

(2) slope - point form.

$$\boxed{y - y_1 = m(x - x_1)}.$$

(3) Two points form.

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}.$$

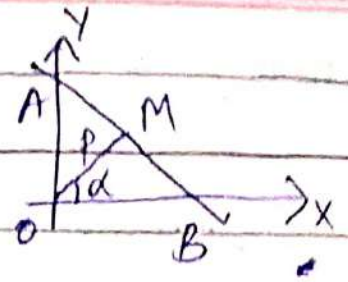
(4) Intercept form.

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}.$$

5) Normal form:-

Let P be the length of perpendicular from the origin to a given line and α be the angle made by this perpendicular with the +ve dirn of x-axis then the eqn of line is given by

$x\cos\alpha + y\sin\alpha = p$.



Note-1 :-

eqn of X-axis is

$y = 0$.

Note-2 :-

eqn of Y-axis is

$x = 0$.

Note-3 :-

The eqn of a line parallel to Y-axis is given

$x = k$ or $x = -k$.

Note-4 :-

eqn of a line parallel to X-axis is

$y = k$ or $y = -k$.

Angle betn two lines:

Let the eqn of lines are $y = m_1x + c_1$, and

$y = m_2x + c_2$

The angle betn these two line is given by

$$\boxed{\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1m_2}\right|}.$$

Note-1:-

The eqn of a line parallel to the given line $ax + by + c = 0$ is, $ax + by + \lambda = 0$.

where λ is a constant.

Note-2:-

The eqn of a line perpendicular to a given line $ax + by + c = 0$ is, $bx - ay + \lambda = 0$.

where λ is a constant.

Point of intersection of two lines:-

The co-ordinates of the point of intersection of the two intersecting lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right).$$

Concurrency of three lines:-

Three lines are said to be concurrent if they meet at a single point. The three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are said to be concurrent if determinant of -

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$$

$$\Rightarrow \boxed{a_1(b_2c_3-b_3c_2) - b_1(a_2c_3-a_3c_2)+c_1(a_2b_3-a_3b_2)}$$

Note:-

Two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are,

(i) coincident, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

(ii) parallel, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

(iii) Perpendicular, if $a_1a_2+b_1b_2=0$.

(iv) Intersecting, if they are neither coincident nor parallel.

Perpendicular distance of a point from a line:-

The perpendicular distance of a point (x_1, y_1) from a line $ax+by+c=0$ is given by

$$\boxed{d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|}.$$

The length of Perpendicular from origin (O) on the line $ax+by+c=0$.

$$\boxed{d = \left| \frac{c}{\sqrt{a^2+b^2}} \right|}$$

Note :-

Let $a_1x+b_1y+c_1=0$ and $a_1x+b_1y+c_2=0$ are two parallel lines.

Distance betn them is given by

$$\boxed{d = \left| \frac{c_1-c_2}{\sqrt{a_1^2+b_1^2}} \right|}$$

(Q) If the distance betn the point $(3,a)$ and $(6,1)$ is 5. Find the value of a.

$|PQ| = \sqrt{(6-3)^2+(1-a)^2}$

$\Rightarrow 5 = \sqrt{9+1+a^2-2a}$

$\Rightarrow 5 = \sqrt{10+a^2-2a}$

$\Rightarrow 25 = a^2-2a+10$

$\Rightarrow a^2-2a = 25-10 = 15.$

$\Rightarrow a(a-2) = 15.$

$\Rightarrow a^2-2a-15 = 0.$

$\Rightarrow a^2-5a+3a-15=0$

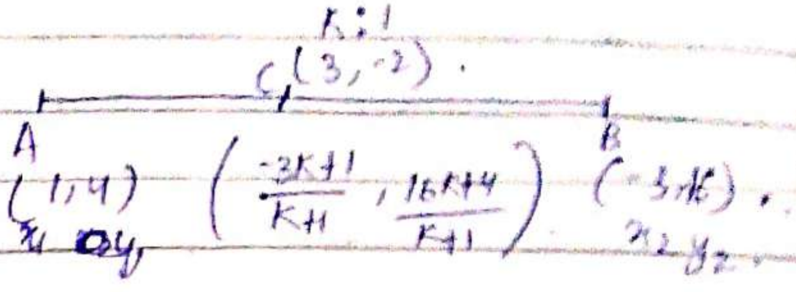
$\Rightarrow a(a-5)+3(a-5)=0.$

$\Rightarrow (a-5)(a+3)=0.$

$\Rightarrow a-5=0$ or $a+3=0.$

(Q) In what ratio does the point (3, -2) divide the line segment ~~join~~ joining the points (1, 4) and (-3, 16).

Ans :-



~~x = 3k 3×(3)~~

let $\frac{-3k+1}{k+1} = 3$

$\Rightarrow -3k+1 = 3k+3.$

$\Rightarrow -3k-3k = 2.$

$\Rightarrow -6k = 2$

$\Rightarrow k = \frac{-2}{6} = \frac{-1}{3}.$

point of C divides the line AB in the ratio 1:3 externally.

$y = \frac{16k+4}{k+1} = -2.$

$\Rightarrow 16k+4 = -2k-2.$

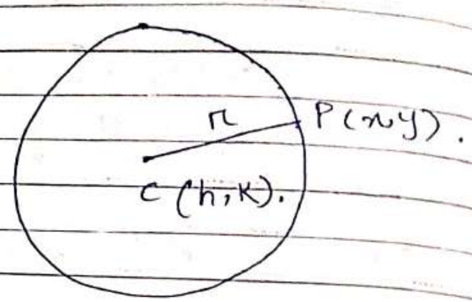
$\Rightarrow 18k = -6.$

$\Rightarrow k = \frac{-6}{8} = \frac{-1}{3}.$

so the ration $(m:n) = (1:3)$.

CIRCLE

A circle is the LOCUS of a point which moves in a plane in such a way that its distance from a fixed point is always constant.
The fixed point is called the center of the circle and the constant distance is called radius.



Equation of circle (center radius form):-

The eqn of a circle whose centre is (h,k) whose radius is r is given by

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}.$$

Proof:-

$$CP = \sqrt{(x-h)^2 + (y-k)^2}$$

$$\Rightarrow r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$\Rightarrow r^2 = (x-h)^2 + (y-k)^2.$$

General eqn of a circle:-

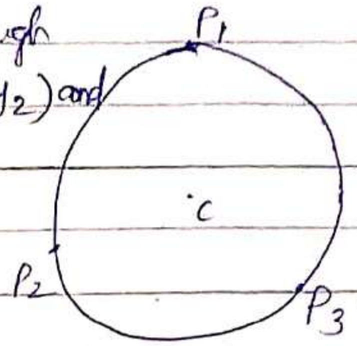
→ The general eqn of ~~the~~ a circle is of the form $\boxed{x^2+y^2+2gx+2fy+c=0}$. where g, f, c are constant.

$\boxed{\text{centre } (-g, -f)}$.

$\boxed{\text{radius}(r) = \sqrt{g^2+f^2-c}}$.

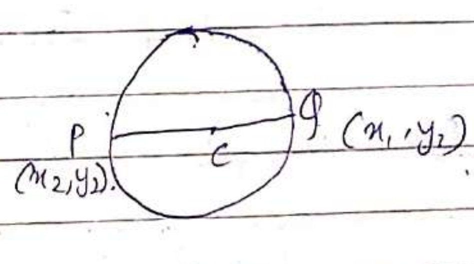
Eqn of a circle passing through three given point:-

The eqn of a circle Passing through three point (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by



$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix}.$$

Eqn of a circle when two end Points of a diameter are given :-



The eqn of a circle having two ends of the diameter are (x_1, y_1) and (x_2, y_2) is given by $\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$.

Orthogonal circles:-

Two circles are said to intersect orthogonally if there angle of intersection is $90°$.

let $S_1 = x^2+y^2+2g_1x+2f_1y+c_1=0$.

and $S_2 = x^2+y^2+2g_2x+2f_2y+c_2=0$.

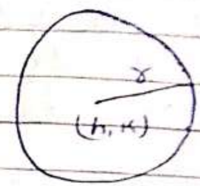
will be orthogonal if,

$$2g_1g_2 + 2f_1f_2 - c_1 - c_2 = 0.$$

Dt 24.12.21

(Q.) Find the eqn of the circle whose centre is (-2, 4) and radius is 9.

Ans:- $(x-h)^2 + (y-k)^2 = r^2.$



$\Rightarrow (x+2)^2 + (y-4)^2 = 9^2.$

$\Rightarrow x^2 + 4x + 4 + y^2 - 8y + 16 = 81$

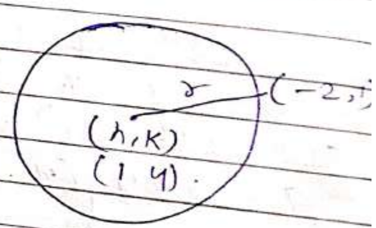
$\Rightarrow x^2 + y^2 + 4x - 8y + 20 = 81$

$\Rightarrow x^2 + y^2 + 4x - 8y - 61 = 0.$

(Q.) Find the eqn of the circle whose centre is at (1, 4) and passing through (-2, 1).

Ans:- $r = \sqrt{(-2-1)^2 + (1-4)^2}$

$= \sqrt{9+9}$



$= \sqrt{18} = 3\sqrt{2}.$

the eqn of circle is

$(x-1)^2 + (y-4)^2 = r^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 8y + 16 = (3\sqrt{2})^2$

$\Rightarrow x^2 - 2x + y^2 - 8y + 17 = 18$

$\Rightarrow x^2 + y^2 - 2x - 8y - 1 = 0.$

Co-ordinate Geometry in 3 Dimension (3D) 4.1.22

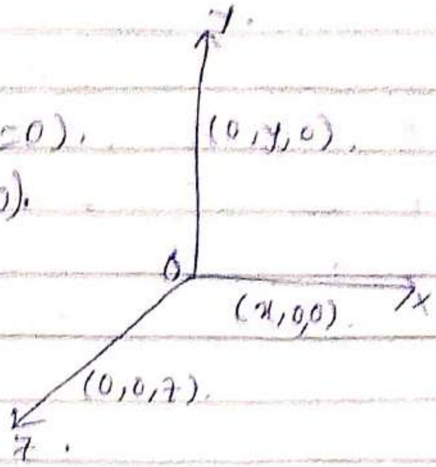
Let x-axis, y-axis, z-axis represents a 3-D Structure.

In this we have three planes

xy-plane (z-co-ordinate = 0).

yz-plane (x-co-ordinate = 0).

xz-plane (y-co-ordinate = 0).



Distance betn two points (Distance formula):-

The distance betn the point $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in this planes.

$$\boxed{|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

Division formula:-

Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points and point R divides PQ in the ratio m : n.

The co-ordinates of R $\boxed{\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)}$.

★ If ratio is not given we assume R divides PQ in the ratio K : 1.

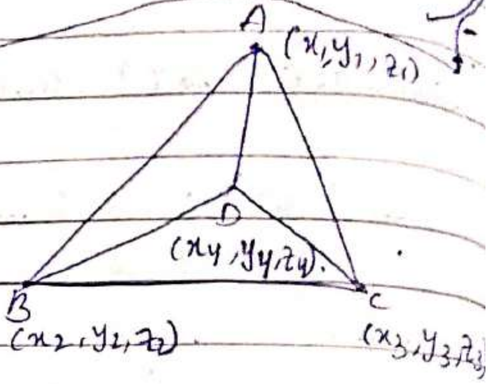
Centroid of a triangle:-

The centroid of the triangle with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is given by, $\boxed{\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)}$.

Centroid of a Tetrahedron :-

The centroid of the tetrahedron with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ is given by,

$$\boxed{\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)}$$



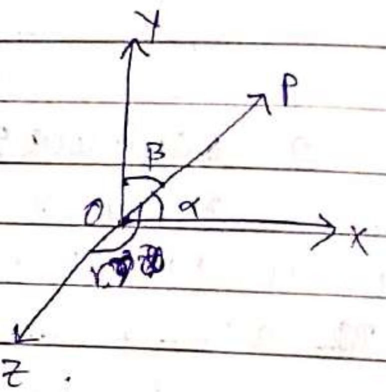
Dir^n cosine :- (D.C.S)

D.C.S, l, m, n.

$l = \cos\alpha$.

$m = \cos\beta$.

$n = \cos\gamma$



Dir^n ratios :- (D.R.S)

Suppose we have three numbers a, b, c which is ~~Propotio~~ Propotional to the direction cosine of a line AB. These numbers a, b, c are called dirn ratios of the line AB and it is denoted by

$\boxed{\langle a, b, c \rangle}$

If (x_1, y_1, z_1) and (x_2, y_2, z_2) are two end points of line then the dirn ratios of these line is given by

$$\boxed{\begin{aligned} a &= x_2 - x_1 \\ b &= y_2 - y_1 \\ c &= z_2 - z_1 \end{aligned}}$$

Relation betn D.C.s and D.R.s :-

(1) $\boxed{l^2+m^2+n^2=1}$.

(2) $$\boxed{l=\frac{a}{\sqrt{a^2+b^2+c^2}},\quad m=\frac{b}{\sqrt{a^2+b^2+c^2}},\quad n=\frac{c}{\sqrt{a^2+b^2+c^2}}}$$

Angle betn two lines:-

The angle betn two lines having dirn cosines l_1, m_1, n_1 and l_2, m_2, n_2 is given by

$$\boxed{\theta = \cos^{-1}\left(l_1l_2+m_1m_2+n_1n_2\right)}.$$

The angle betn two lines by using dirn ratios

$$\boxed{\theta = \cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)}.$$

Note :-

(1) Two lines are said to be perpendicular to each other if $\theta = 90^\circ$.

$$\theta = \cos^{-1}(l_1l_2+m_1m_2+n_1n_2).$$

$$\Rightarrow \cos 90^\circ = l_1l_2+m_1m_2+n_1n_2$$

$$\Rightarrow \boxed{l_1l_2+m_1m_2+n_1n_2=0}.$$

or

$$\boxed{a_1a_2+b_1b_2+c_1c_2=0}.$$

(2) Two lines are said to be Parallel if.

$$\boxed{\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}}.$$

Or

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}.$$

Projection of a line segment :-

The projection of a line segment Joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on a line with dirn cosine l, m, n is ~~given~~ given by

$$\boxed{l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1) = 0}.$$

Q) Find the distance betn the points $P(2, 3, 5)$ and $Q(4, 3, 1)$. Dt. 5.1.2

$x_1 = 2$, $y_1 = 3$, $z_1 = 5$

$x_2 = 4$, $y_2 = 3$, $z_2 = 1$

Ans :- $|PQ| = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

$= \sqrt{4 + 0 + 16} = \sqrt{20} = 2\sqrt{5}$.

Q) - Find the value of x if the distance betn the two points $(x, -8, 4)$ and $(3, -5, 4)$ is 5

Ans :- $\sqrt{(3-x)^2 + (-5+8)^2 + (4-4)^2} = 5$.

$\Rightarrow \sqrt{(3-x)^2 + 9 + 0} = 5$

$\Rightarrow \sqrt{9 - 6x + x^2 + 9} = 5$

$\Rightarrow \quad x^2 - 6x + 18 - 25 = 0$

$\Rightarrow \quad x^2 - 6x - 7 = 0$

$\Rightarrow \quad x^2 - 7x + x - 7 = 0$

$\Rightarrow \quad x(x-7) + 1(x-7) = 0$

$\Rightarrow \quad x = 7 \text{ or } x = -1$

(Q) - Find the co-ordinates of a point which divides the point (1, 3, 7) and (6, 3, 2) in the ratio 2 : 3.

Ans :- P (1, 3, 7). R(2:3). Q (6, 3, 2).

$$R\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

$$= \left(\frac{12+3}{2+3}, \frac{6+9}{2+3}, \frac{4+21}{2+3}\right)$$

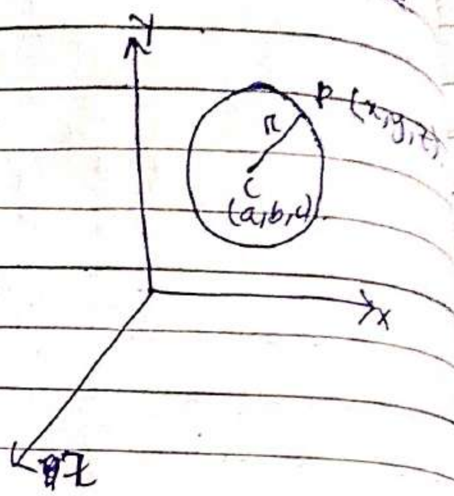
$$= \frac{15}{5}, \frac{15}{5}, \frac{25}{5}$$

$$= (3, 3, 5)$$

$\therefore R(3, 3, 5)$.

SPHERE

A sphere is the locus of a point in a space which moves in such a way that it remains always at a constant distance from is fixed point. The fixed point is called the centre and the constant distance is called radius of the sphere.



NOTE-1:-

→ The eqn of the sphere with center at the point (a, b, c) and radius 'r' is given by

$$\boxed{(x-a)^2+(y-b)^2+(z-c)^2=r^2}.$$

General eqn of a sphere :-

→ It is given by $\boxed{x^2+y^2+z^2+2ux+2vy+2wz+d=0}$.

$$\boxed{\text{centre } (-u, -v, -w)}.$$

$$\boxed{\text{radius} = \sqrt{u^2+v^2+w^2-d}}.$$

NOTE-2:-

→ The eqn of the sphere ~~with~~ with the end points of a diameter of a sphere is given by

$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0}.$$

NOTE-3 :-

The eqn of the sphere passing through 4 given points.
(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) is obtained by solving the general equation of the sphere.

(Q) Find the centre and radius of the sphere
$4x^2+4y^2+4z^2-16x-24z+3=0$.

Ans:- $4x^2+4y^2+4z^2-16x-24z+3=0$.

$\Rightarrow \frac{4x^2}{4}+\frac{4y^2}{4}+\frac{4z^2}{4}-\frac{16x}{4}-\frac{24}{4}z+\frac{3}{4}=0$

$\Rightarrow x^2+y^2+z^2-4x-6z+\frac{3}{4}=0$.

$2u=-4$, $2v=0$, $2w=-6$, $d=\frac{3}{4}$.
$\Rightarrow u=-2$. $\Rightarrow v=0$ $\Rightarrow w=-3$.

centre $=(2, 0, 3)$.

~~Radius~~

radius $=\sqrt{(-2)^2+0^2+(-3)^2-\frac{3}{4}}$.

$=\sqrt{4+9-\frac{3}{4}}$

$=\sqrt{\frac{16+36-3}{4}}$

$=\sqrt{\frac{49}{4}}=\frac{7}{2}$.

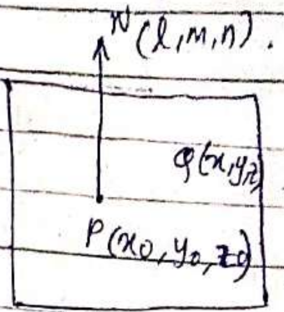
Plane

Dt. 7.1.22

NOTE-1:-

→ Eqn of a plane passing through a point $P(x_0, y_0, z_0)$ when the dirn cosine of its normal are $N(l, m, n)$ is given by ~~$l(x-x_0)$~~

$$\boxed{l(x-x_0)+m(y-y_0)+n(z-z_0)=0}$$



NOTE-2:-

→ If dirn ratios are given i.e $\langle a, b, c\rangle$ and passing through the point (x_0, y_0, z_0) then the eqn of plane is

$$\boxed{a(x-x_0)+b(y-y_0)+c(z-z_0)=0}$$

NOTE-3:-

→ The general eqn of plane is

$$\boxed{ax+by+cz+d=0}$$ where a, b, c, d are const.

NOTE-4:-

→ The eqn of the plane passing through 3 non-colinear points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

NOTE-5 :-

→ The eqn of the plane in intercept form is

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

. where a, b, c are the intercept on the axis respectively.

Eqn of plane parallel to co-ordinate axis :-

→ Eqn of plane parallel to x-axis is

$\boxed{by+cz+d=0}$.

→ Eqn of plane parallel to y-axis is

$\boxed{ax+cz+d=0}$.

→ Eqn of plane parallel to z-axis

$\boxed{ax+by+d=0}$.

Eqn of plane perpendicular to co-ordinate axis :-

→ Eqn of plane perpendicular to x-axis is

$\boxed{x=K}$. when K is any const.

→ Eqn of plane perpendicular to y-axis is

$\boxed{y=K}$.

→ Eqn of plane perpendicular to z-axis is

$\boxed{z=K}$.

Eqn of Plane passing through the Intersection of 2 Planes

Let the eqn of two planes are

$$a_1x+b_1y+c_1z+d_1=0.$$

and $a_2x+b_2y+c_2z+d_2=0.$

The eqn of plane is given by

$$\boxed{a_1x+b_1y+c_1z+d_1+K(a_2x+b_2y+c_2z+d_2)}$$

Angle betn two planes:-

Let the two planes are

$$a_1x+b_1y+c_1z+d_1=0.$$

and $a_2x+b_2y+c_2z+d_2=0.$

Let θ be the angle betn two planes. i.e.

$$\boxed{\theta=\cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)}.$$

Two planes are perpendicular

If $\theta=90^\circ$, then

$$\boxed{a_1a_2+b_1b_2+c_1c_2=0}.$$

Two planes are parallel.

If $$\boxed{\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}}.$$

Two planes are said to be Identical

If $$\boxed{\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}}.$$

Distance of a point from a plane :-

→ The length of the perpendicular from point (x_1, y_1, z_1) to the plane

$ax+by+cz+d=0$ is given by

$$\boxed{d = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|}$$

Distance between two parallel planes :-

Let two parallel planes are

$ax+by+cz+d_1=0$.

and $ax+by+cz+d_2=0$.

$$\boxed{\text{Distance } d = \left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|}$$

(Q) Find the eqn of the plane containing the line of intersection of the planes $x+y+z+1=0$, $2x-3y+5z-2=0$ and passing through the point $(-1, 2, 1)$.

Ans :- $x+y+z+1=0$.

$2x-3y+5z-2=0$.

The eqn of point,

$(x+y+z+1)+k(2x-3y+5z-2)=0$.

$\Rightarrow ((-1)+2+1+1)+k(-2-6+5-2)=0$

$\Rightarrow 3-5k=0$

$\Rightarrow -5k=-3 \Rightarrow k=\frac{3}{5}$.

~~Put the value of k~~

Hence the eqn of plane,

$$(x+y+z+1) + \frac{3}{5}(2x-3y+5z-2)=0.$$

$$\Rightarrow x+y+z+1+\frac{6}{5}x-\frac{9}{5}y+\frac{15z}{5}-\frac{6}{5}=0.$$

$$\Rightarrow 5x+5y+5z+5+6x-9y+15z-6=0.$$

$$\Rightarrow 11x-4y+20z-1=0.$$