MATHEMATICS-1(1st Semester Diploma Engg:-2024-25)
UNIT-I

A. 02 Mark Questions & Solutions *Taxonomy Level*

1. **$Define\ ASTC\ rule$** **Level-1 (Remembering)**
Answer $ASTC = All, Sine, Tan, Cos$ In the 1st Quadrant All trigonometric ratios are positive, in 2nd Quadrant only sin and cosec are positive, in 3rd Quadrant tan and cot are positive and in 4th Quadrant only Cos and Sec are positive.
2. **Convert 660° into radian** **Level-1 (Remembering)**

Answer: - 1 degree$= \frac{\pi}{180} R \Longrightarrow 660° = \frac{\pi}{180} \times 660 = \frac{11\pi}{3}$

3. **Convert $\frac{9\pi}{2}$ into degree** **Level-1(Remembering)**

Answer: - $\pi = 180° => \frac{9\pi}{2} = \frac{9}{2} x180 = 810°$

4. **convert remaining two unit of measurement.(A).30°(B)2^G**
Level-1 (Remembering)
Answer:- $A. We\ know\ \ x^D = \left(\frac{10x}{9}\right)^G \Rightarrow 30^0 = \left(\frac{10\times30}{9}\right)^G = \left(\frac{300}{9}\right)^G$
$and\ also\ x^D = \left(\frac{\pi x}{180}\right)^R$ so, $30^0 = \left(\frac{\pi\times30}{180}\right)^R = \left(\frac{\pi}{6}\right)^R$
B. we know that $x^G = \left(\frac{9x}{10}\right)^D and\ \ x^G = \left(\frac{\pi x}{200}\right)^R$
So,$2^G = \left(\frac{9\times2}{10}\right)^D = \left(\frac{18}{10}\right)^D$
$2^G = \left(\frac{\pi\times2}{200}\right)^R = \left(\frac{\pi}{100}\right)^R$

5. **Evaluate cos 75°** **Level-2 (Understanding)**

Answer: $\cos(45° + 30°) = \cos 45°. \cos 30° - \sin 45°. \sin 30° = \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

6. **Evaluate tan15°** **Level-2 (Understanding)**

Answer:- $\tan 15° = \tan(45° - 30°) = \frac{tan45°-tan30°}{1+tan45°\times tan30°} = \frac{1-(1/\sqrt{3})}{1+(1/\sqrt{3})} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$.
7. **Evaluate tan $(840°)$** **Level-2 (Understanding)**
Answer:- $\tan(840°) = \tan\left(9 \times \frac{\pi}{2} + 30°\right) = -cot30° = -\sqrt{3}$

8. **Find the value of sin 1485** **Level-2 (Understanding)**

Answer $\sin 1485 = \sin(4 * 360 + 45) = \sin 45 = \frac{1}{\sqrt{2}}$

9. **Prove that tan660°.cot1320°+cot390°.tan210°=0**

Level-2 (Understanding)

Answer: - tan660°.cot1320°+cot390°.tan210

= tan (360°+300°).cot (3x360°+240°) +cot (360°+30°).tan210°

= tan 300°. Cot 240° + cot 30°. tan 210°

= tan (270°+30°). Cot (270°-30°) + cot 30°. tan (180°+30°)

= -cot 30°. tan 30° + cot 30°. tan 30°

= 0 (Proved)

10. **Find the value of cos1°.cos2°……..cos100°.** **Level-2 (Understanding)**

Answer: - cos1°.cos2°……..cos100°

=cos1°.cos2°……..cos90°…..cos100°

=0 [as cos90°=0]

11. **Find the value of $\sec 3660^{\circ}$.** **Level-2 (Understanding)**

Ans:

$\cos 3660^{\circ} = \cos(360^{\circ} \times 10 + 60^{\circ}) = \cos 60^{\circ} = \frac{1}{2}$

$\therefore \quad \sec 3660^{\circ} = \frac{1}{\cos 3660^{\circ}} = \frac{1}{1/2} = 2 \quad$ (Ans)

12. **Find the value of cos 24°+cos 5°+cos 175°+cos 204°+cos 300° L2(Understanding)**

Answer: - cos 24°+cos 5°+cos 175°+cos 204°+cos 300

= cos 24°+cos 5°+cos (180°-5°)+cos (180°+24°)+cos(90 x3°+ 30°)

= cos 24°+cos 5°-cos 5°-cos 24°+sin30°

= sin30° =1/2

13. **find the value of tan1°.tan2°……tan89°** **Level-2 (Understanding)**

Answer: - tan1°.tan2°……tan89°

=tan1°.tan2°……tan44°.tan45°.tan46°…….tan88°.tan89°

= tan1°.tan2°……tan44°.1. tan(90°-44°)…….tan(90°-2°).tan(90°-1°)

= tan1°.tan2°……tan44°.1.cot44°……..cot2°.cot1°

= (tan1°.cot1°).(tan2°.cot2°)……..(tan44°.cot44°).1

=1.1.1……….1

=1

14. **Find the value of $\cos 20°\,.\,\cos 40°\,.\,\cos 80°$** **Level-2 (Understanding)**

Answer: $\cos 20°\,.\cos 40°\,.\cos 80°$

$= \cos 20°\,.\cos(60° - 20°)\,.\cos(60° + 20°)$

$= \frac{1}{4}.\cos 3 \times 20° = \frac{1}{4}.\cos 60° = \frac{1}{4}.\frac{1}{2} = \frac{1}{8}$

15. **Prove that $\sqrt{\frac{1-cos2A}{2}} = \pm \sin A$** **Level-1(Remembering)**

Answer: - $\sqrt{\frac{1-cos2A}{2}} = \sqrt{\frac{2\sin^2 A}{2}} = \pm\, Sin\ A$

16. **Prove that $\tan 56° = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$** **Level-2 (Understanding)**

Answer: $\tan 56° = \tan(45° + 11°) = \frac{\tan 45° + \tan 11°}{1 - \tan 45°.\tan 11°}$

$= \frac{1+\frac{\sin 11°}{\cos 11°}}{1-\frac{\sin 11°}{\cos 11°}} = \frac{\frac{\cos 11° + \sin 11°}{\cos 11°}}{\frac{\cos 11° - \sin 11°}{\cos 11°}} = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

17. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, *evaluate* $\tan(A + B)$. Level-2(Understanding)

Answer:- $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$

18. Evaluate $2\sin 75° \times \sin 15°$. Level-2(Understanding)

Answer:- $2\sin 75° \times \sin 15° = 2\sin(90° - 15°)\ \sin 15° =$

$2\cos 15° \sin 15° = \sin 30° = \frac{1}{2}$

19. Evaluate $\sin(45° + \theta) - \cos(45° - \theta)$ Level-2(Understanding)

Answer:- $\sin(45° + \theta) - \cos(45° - \theta)$

$= \sin(90° - (45° - \theta)) - \cos(45° - \theta)$

$= \cos(45° - \theta) - \cos(45° - \theta)$

$= 0$.

20. Find A , if $\frac{1+tanA}{1-tanA} = \sqrt{3}$. Level-2 (Understanding)

Answer:- $\frac{1+tanA}{1-tanA} = \frac{tan45°+tanA}{1-tan45°tanA} \Rightarrow \tan(45° + A) = \sqrt{3}$

$\Rightarrow \tan(45° + A) = tan60°$

$\Rightarrow A = 15°$.

21. Find $\sin 35° + \cos 5°$. Level-2(Understanding)

Answer:- $\sin 35° + \cos 5°$

$= \sin 35° + \cos(90 - 85°)$

$= \sin 35° + \sin 85°$

$= 2\ \sin(\frac{35°+85°}{2})\cos(\frac{85°-35°}{2})$

$= 2\sin 60° \cos 25°$

$= 2\frac{\sqrt{3}}{2}\cos 25° = \sqrt{3}\cos 25°$.

22. Find $\sin 70°(4cos^2 20° - 3)$. Level-2 (Understanding)

Answer:- $\sin 70°(4cos^2 20° - 3)$

$= \sin(90° - 20°)\ (4cos^2 20° - 3)$

$= \cos 20°(4cos^2 20° - 3)$

$= 4cos^3 20°$ -3$\cos 20°$

$= \cos 3 \times 20° = \cos 60° = ½$

23. Evaluate $\frac{\tan 15°}{1-tan^2 15°}$. Level-2 (Understanding)

Answer:- $\frac{\tan 15°}{1-tan^2 15°} = \frac{2\tan 15°}{2(1-tan^2 15°)}$

$= \frac{tan30°}{2} = (1/\sqrt{3})/2 = \frac{1}{2\sqrt{3}}$

24. Evaluate $sec^2 42° - cosec^2 48°$. Level-2 (Understanding)

Answer:- $sec^2 42° - cosec^2 48°$

$= sec^2 42° - cosec^2(90° - 42°)$

$= sec^2 42° - sec^2 42°$

$= 0$

25. $\boldsymbol{If\ \cos A = \frac{\sqrt{3}}{2}\ then\ find\ \cos 3A}$ **Level-2 (Understanding)**

Answer $\cos 3A = 4\cos^3 A - 3\cos A = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\frac{\sqrt{3}}{2} = 4 * 3\frac{\sqrt{3}}{8} - 3\frac{\sqrt{3}}{2} = 0$

26. $Prove\ that\ \frac{\cos 17+\sin 17}{\cos 17-\sin 17} = \tan 62$

Answer $\frac{\cos 17+\sin 17}{\cos 17-\sin 17} = \frac{1+\tan 17}{1-\tan 17} = \frac{\tan 45+\tan 17}{1-\tan 45 . \tan 17} = \tan(45+17) = \tan 62$

27. $\boldsymbol{If\ tanx + tany = 5\ and\ tanx.tany = \frac{1}{2}, then \cot(x+y) = ?}$

L2 (Understanding)

Answer$\cot(x+y) = \frac{1}{\tan(x+y)} = \frac{1}{\frac{tan+tany}{1-tanx.tany}} = \frac{1-tanx.tany}{tanx+tany} = \frac{1-\frac{1}{2}}{5} = \frac{1}{10}$

28. If $\boldsymbol{\tan\theta = \frac{1}{\sqrt{7}}}$ **and** $\boldsymbol{0 < \theta < \frac{\pi}{2}}$, **find the value of** $\boldsymbol{\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}}$

Level-2 (Understanding)

Ans:

Given $\tan\theta = \frac{1}{\sqrt{7}} = \frac{p}{b}$

As per the Pythagoras theorem, $p^2 + b^2 = h^2$

Or, $h = +\sqrt{p^2+b^2} = \sqrt{1^2 + (\sqrt{7})^2} = \sqrt{8}$

$\therefore p = 1, b = \sqrt{7}$ and $h = \sqrt{8}$

Then, $\operatorname{cosec}\theta = \frac{h}{p} = \frac{\sqrt{8}}{1} = \sqrt{8}$ and $\sec\theta = \frac{h}{b} = \frac{\sqrt{8}}{\sqrt{7}} = \sqrt{\frac{8}{7}}$.

Now, $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(\sqrt{8})^2 - \left(\sqrt{\frac{8}{7}}\right)^2}{(\sqrt{8})^2 + \left(\sqrt{\frac{8}{7}}\right)^2} = \frac{8-\frac{8}{7}}{8+\frac{8}{7}} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}} = \frac{48}{64} = \frac{3}{4}$

29. If $\boldsymbol{\sin A = \frac{3}{5}}$ **and** $\boldsymbol{\cos B = \frac{12}{13}}$, **where** $\boldsymbol{A}$ **and** $\boldsymbol{B}$ **be the acute angles,**

Then find the value of $\boldsymbol{\cos(A+B)}$. **Level-2(Understanding)**

Ans:

Given $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$

Now, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \sqrt{1-\sin^2 A}\ \cos B - \sin A\ \sqrt{1-\cos^2 B}$

$= \sqrt{1-\left(\frac{3}{5}\right)^2}\ \frac{12}{13} - \frac{3}{5}\sqrt{1-\left(\frac{12}{13}\right)^2}$

$= \frac{4}{5}\ \frac{12}{13} - \frac{3}{5}\ \frac{5}{13} = \frac{48-15}{65} = \frac{33}{65}$ (Ans)

30. If $\tan A = \frac{5}{6}$ and $\tan B = \frac{1}{11}$, find the value of $A + B$ in radian.

L-2 (Understanding)

Ans:

Given $\tan A = \frac{5}{6}$ and $\tan B = \frac{1}{11}$,

Now, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \ \tan B} = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \cdot \frac{1}{11}} = \frac{61/66}{61/66} = 1 = \tan\frac{\pi}{4}$

Or, $A + B = \frac{\pi}{4}$ (Ans)

31. Find the value of $\sin 22\frac{1}{2}^{\circ}$. **Level-3 (Applying)**

Ans:

We know that, $\sin\theta = \pm\sqrt{\frac{1}{2}(1 - \cos 2\theta)}$

Put $\theta = 22\frac{1}{2}^{\circ}$

$\therefore \quad \sin 22\frac{1}{2}^{\circ} = \pm\sqrt{\frac{1}{2}(1 - \cos 45^{\circ})} = \pm\sqrt{\frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)} = \pm\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} = \pm\sqrt{\frac{2-\sqrt{2}}{4}}$

$\Rightarrow \sin 22\frac{1}{2}^{\circ} = \pm\frac{\sqrt{2-\sqrt{2}}}{2}$

Since $22\frac{1}{2}^{\circ}$ lies in 1st quadrant

and as per ASTC rule $\sin 22\frac{1}{2}^{\circ}$ has a positive value,

$\therefore \sin 22\frac{1}{2}^{\circ} = \frac{\sqrt{2-\sqrt{2}}}{2}$ (Ans)

32. Find the value of $\frac{1 - \tan 15^{\circ}}{1 + \tan 15^{\circ}}$. **Level-2 (Understanding)**

Ans:

$\frac{1 - \tan 15^{\circ}}{1 + \tan 15^{\circ}} = \frac{\tan 45^{\circ} - \tan 15^{\circ}}{1 + \tan 45^{\circ} \tan 15^{\circ}} = \tan(45^{\circ} - 15^{\circ}) = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$
(Ans)

33. Find the value of $2\sin 105^{\circ} \ \sin 15^{\circ}$ **Level-2 (Understanding)**

Ans:

$2\sin 105^{\circ} \ \sin 15^{\circ}$

$= 2\sin(90^{\circ} + 15^{\circ}) \ \sin 15^{\circ}$

$= 2\cos 15^{\circ} \ \sin 15^{\circ}$

$= \sin 30^{\circ}$ $[\because \ 2\sin\theta\cos\theta = \sin 2\theta]$

$= \frac{1}{2}$ (Ans)

34. **Find the value of $\sin 20^{\circ}\,(3-4\cos^2 70^{\circ})$.** **Level-2 (Understanding)**

Ans:

$\sin 20^{\circ}\,(3-4\cos^2 70^{\circ})$

$= \sin 20^{\circ}\,[3-4\cos^2(90^{\circ}-20^{\circ})]$

$= \sin 20^{\circ}\,(3-4\sin^2 20^{\circ})$

$= 3\sin 20^{\circ}-4\sin^3 20^{\circ}$

$=\sin(3\times 20^{\circ})$

$=\sin 60^{\circ}$

$=\frac{\sqrt{3}}{2}$ (Ans)

35. **Find the minimum value of $\sin\theta\ \cos\theta$.** **Level-2 (Understanding)**

Ans:

Minimum Value of $\sin\theta\ \cos\theta$

$=$ Minimum Value of $\frac{1}{2}(2\ \sin\theta\ \cos\theta)$

$=$ Minimum Value of $\frac{1}{2}\sin 2\theta$

$= \frac{1}{2}(-1) =-\frac{1}{2}$ (Ans)

36. **Find the value of $\mathbf{cosec^2\left(\frac{7\pi}{6}\right)}$.** **Level-2 (Understanding)**

Answer:-$\text{cosec}^2\frac{7\pi}{6}=\left(co\sec\left(\pi+\frac{\pi}{6}\right)\right)^2=\left(-\text{ cosec}\frac{\pi}{6}\right)^2=(-2)^2=4$

37. **Evaluate $\cos^2\left(\frac{\pi}{4}+x\right)-\sin^2\left(\frac{\pi}{4}-x\right)$** **Level-2 (Understanding)**

Answer:-$\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B$

so,let $\frac{\pi}{4}+x=A\ and\ \frac{\pi}{4}-x=B$

$=\cos\left(\frac{\pi}{4}+\varkappa+\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}+x-\frac{\pi}{4}+x\right)$

$=cos\frac{\pi}{2}\cdot cos\,2x$

$=0\times\cos 2x$=0

38. **If $\sec\theta=\frac{5}{4}$ then find $\tan\frac{\theta}{2}$?** **Level-2 (Understanding)**

Answer:-$\sec\theta=\frac{5}{4}$, $\cos\theta=\frac{4}{5}$

$Now\ \tan\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$=\pm\sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}}= \quad =\pm\sqrt{\frac{\frac{1}{5}}{\frac{9}{5}}}=\pm\sqrt{\frac{1}{9}}=\frac{1}{3}$

39. **Find the value of (cos50 °cos40° -sin50°sin40°)?** **Level-2 (Understanding)**

Answer:-$\cos(50^0+40^0)=\cos 90^0=0$

40. **Find the value of $cos\,20^0(3-4\,sin^2\,70^0)$.** **Level-(Understanding)**

Answer:- $cos\,20^0(3-4\,sin^2\,70^0)$

$=cos\,20^0\left(3-4\,sin^2\,(90^0-20^0)\right)$

$= cos\, 20^0 (3 - 4\, cos^2\, 20^0)$

$= 3\, cos\, 20^0 - 4\, cos^3\, 20^0$

$= -(\cos 3 \times 20^0) = -\, cos\, 60^0 = -\, \frac{1}{2}$

41. If $\sin \theta = \frac{24}{25}$ and θ *lies in 2nd quadrant* then find $\sec \theta + \tan \theta$

L-3(Applying)

Answer:-Here P=24 And h=25 , So applying Pythagoras theorem b=$\sqrt{25^2 - 24^2} = \pm 7$

Given that Θ lies in 2nd quadrant sec and tan both are negative.

So $\sec \theta = \frac{-25}{7}$ *and* $\tan \theta = \frac{-24}{7}$

So , $\sec\ \theta + \tan\ \theta =$ -7

42. Find the value of cos12°+cos84°+cos156°+cos132°.

Level-2 (Understanding)

Answer:- (cos 12° +cos 132°)+(cos84°+ cos 156 °)

=2cos72°cos60°+2cos120°cos36°

= cos72°- cos36° = sin18°-cos36°= -1/2

43. Find $2 \sin 67\frac{1^0}{2} \cos 22\frac{1^0}{2}$. **Level-2 (Understanding)**

Answer:- $2 \sin 67\frac{1^0}{2} \cos 22\frac{1^0}{2}$.

$= 2 \sin \left(90^0 - 22\frac{1^0}{2}\right) \cos 22\frac{1^0}{2}$

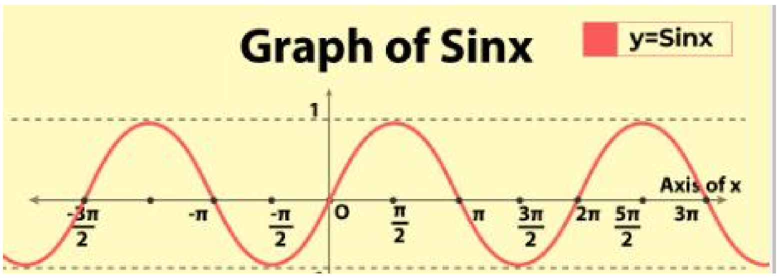
$= 2\cos 22\frac{1^0}{2} \cos 22\frac{1^0}{2} = 2\cos^2 22\frac{1^0}{2}$

$= 1 + \cos 2 \times 22\frac{1^0}{2} = 1 + \cos 45^0 = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}}$

44. Sketch the graph of $\sin x$. **Level-2 (Understanding)**

Ans:

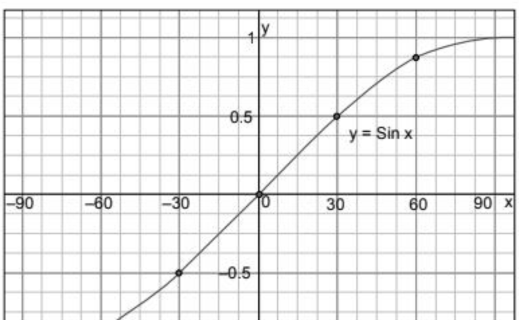
The graph of $\sin x$ is



45. Draw the graph of Y=sinx (-90°< x < 90°) **Level-2 (Understanding)**

Answer:-

X	−60°	−30°	0°	30°	60°
sinx	-0.86	-0.5	0	0.5	0.86



B. 05 Marks Questions & Solutions *Taxonomy Level*

1. **Find the value of $\sin 36° . \sin 72° . \sin 108° . \sin 144°$** **Level-3 (Applying)**

   Answer: - $\sin 36° . \sin 72° . \sin 108° . \sin 144°$

   $= \sin 36° . \sin 144° . \sin 72° . \sin 108°$

   $= \sin 36° . \sin (180° - 36°) . \sin 72° . \sin (180° - 72°)$

   $= \sin 36° . \sin 36° . \sin 72° . \sin 72°$

   $= \sin^2 36° . \sin^2 72°$

   $= \frac{1}{4} 2. \sin^2 36° . 2. \sin^2 72°$

   $= \frac{1}{4}(1 - \cos 2.36°)(1 - \cos 2. 72°)$

   $= \frac{1}{4}(1 - \cos 72°)(1 - \cos 144°)$

   $= \frac{1}{4}(1 - \sin 18°). (1 + \cos 36°)$

   $= \frac{1}{4}\left(1 - \frac{\sqrt{5}-1}{4}\right)\left(1 + \frac{\sqrt{5}+1}{4}\right) = \frac{5}{16}$

2. **Problem- Sin18°** **Level-3 (Applying)**

   Answer: -*Let* θ=18°

   ⟹ 5θ=90°

   ⟹ 3θ+2θ=90°

   ⟹ 2θ=90°-3θ

   ⟹ Sin 2θ = sin (90°-3θ)

   ⟹ Sin 2θ= cos 3θ

   ⟹ 2sinθ.cosθ =4$\cos^3$θ - 3cosθ

   ⟹ 2sinθ.cosθ= cosθ(4$\cos^2$θ-3)

   ⟹ 2sinθ=(4$\cos^2$θ-3)

   ⟹ 2sinθ=4(1-$\sin^2$θ)-3=4-4$\sin^2$θ-3=1-4$\sin^2$θ

   ⟹ 2sinθ=1-4$\sin^2$θ

   ⟹ 2sinθ+4$\sin^2$θ-1=0

   ⟹4$\sin^2$θ+2sinθ-1=0

   Ax^2 + bx +c =0 x= $\frac{-b \mp \sqrt{b^2-4.a.(c)}}{2.a}$

   A=4,b=2,c= -1

   Sinθ=$\frac{-2 \mp \sqrt{2^2-4.4.(-1)}}{2.4} = \frac{-2 \mp \sqrt{20}}{2.4} = \frac{-2 \mp 2\sqrt{5}}{2.4} = \frac{-1 \mp \sqrt{5}}{4}$

   Sin 18° =$\frac{-1 \mp \sqrt{5}}{4}$

   Therefore Sin 18°=$\frac{-1+\sqrt{5}}{4}$ (sin 18 is always +ve)

3. **If sinθ + cosec θ = 2 . show that $\sin^n \theta + cosec^n\theta = 2$ for all positive integers n.**

   Level-2 (Understanding)

   Answer: -sinθ + cosec θ = 2

   ⟹sinθ + $\frac{1}{\sin\theta}$ =2

$\Rightarrow \sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow \sin^2\theta - 2 \,.\, 1 \,.\, \sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0$

$\text{Sin}\theta = 1$, $\text{cosec}\,\theta = 1$

$\sin^n\theta + cosec^n\theta = 1^n + 1^n = 1 + 1 = 2$

4. **If A+B+C=π then prove that Sin 2A+sin 2B+sin 2C= 4 sinA.sinB.sinC**

Level-3 (Applying)

Proof:-

L.H.S

Sin 2A+Sin 2B+Sin 2C

$= 2\sin\frac{2A+2B}{2} \,.\, \cos\frac{2A-2B}{2} + 2\sin C.\,\text{Cos}C$

=2 Sin (A+B).cos(A-B)+2 SinC.cosC

[Given A+B+C=π => C=π-(A+B) & A+B= π-C]

=2 Sin (π-C).cos(A-B)+2 SinC. cos[π-(A+B)]

=2 Sin C.cos(A-B)+2 Sin C.[-cos(A+B)]

=2 Sin C.[Cos(A-B)-Cos(A+B)]

= 2 Sin C.2 Sin A. Sin B

= 4 Sin A. Sin B. Sin C

5. **Find maximum and minimum value of $5\sin x + 12\cos x$.**

Level-3 (Applying)

Answer:- let $5 = r\cos\theta$, $12 = r\sin\theta$

$5^2 = r^2cos^2\theta \quad 12^2 = r^2cos^2\theta$

$\Rightarrow r^2 = 25 + 144 = 169$

$\Rightarrow r = 13$

$5\sin x + 12\cos x.$

$= 13\cos\theta\, sinx + 13\sin\theta\, cosx$

$= 13\sin(\theta + x)$

Maximum value is $13 \times 1 = 13$

Minimum value is $13 \times (-1) = -13$.

6. **If $A + B = 45°$, $\boldsymbol{Prove\ that}\ (1 + tanA)(1 + tanB) = 2$**

Level-3 (Applying)

Answer:- $A + B = 45°$

Tan(A+B) = tan45°

$\Rightarrow \frac{tanA + tanB}{1 - tanAtanB} = 1$

$\Rightarrow tanA + tanB = 1 - tanAtanB$

$\Rightarrow tanA + tanB + tanAtanB = 1$

$\Rightarrow tanA + tanB + tanAtanB + 1 = 1 + 1$

$\Rightarrow (1 + tanA)(1 + tanB) = 2.$

7. $\boldsymbol{Prove\ that\ Cosec\ A - 2\cot 2A.\ \cos A = 2\sin A}$

Level-2 (Understanding)

Answer: -$Cosec\ A - 2\cot 2A.\ \cos A = cosec\ A - 2\frac{\cos 2A}{\sin 2A}\cos A$

$= Cosec\, A - 2\frac{2\cos^2 A - 1}{2\sin A . \cos A}\cos A$

$= \frac{1}{\sin A} - \frac{2\cos^2 A - 1}{\sin A}$

$= \frac{1}{\sin A}[1 - 2\cos^2 A + 1]$

$= \frac{1}{\sin A}[2(1 - \cos^2 A)]$

$= \frac{1}{\sin A} . 2\sin^2 A = 2\sin A$

8. **Prove that $\sin 3A = 3\sin A - 4\sin^3 A$ Level-3 (Applying)**

Answer: -Sin3A= sin(A+2A)= sinA.cos2A+cosA.sin2A

=sinA.($\cos^2$A $-$ $\sin^2$A)+cosA.2sinA.cosA

=sinA(1-$\sin^2$A $-$ $\sin^2$A)+2sinA. $\cos^2$A

=sinA-2$\sin^3$A +2sinA(1-$\sin^2$A)

=sinA-2$\sin^3$A+2sinA-2$\sin^3$A

=3sinA-4$\sin^3$A

9. **Prove that $\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = cosec\theta - cot\theta$ Level-3 (Applying)**

Proof:

L.H.S.

$= \sqrt{\frac{1 - cos\,\theta}{1 + cos\,\theta}}$

$= \sqrt{\frac{1 - cos\,\theta}{1 + cos\,\theta} . \frac{1 - cos\,\theta}{1 - cos\,\theta}}$ [∵ Divide the num. and denom. By $1 - cos\,\theta$]

$= \sqrt{\frac{(1 - cos\,\theta)^2}{1^2 - cos^2\theta}}$

$= \sqrt{\frac{(1 - cos\,\theta)^2}{sin^2\theta}}$

$= \frac{1 - \cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= cosec\theta - \cot\theta$

= R.H.S. (Proved)

10. **Prove that $sin^6\theta + cos^6\theta = 1 - 3\, sin^2\theta\, cos^2\theta$ Level-3 (Applying)**

Proof:

L.H.S.

$= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)\left[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2\right]$ [∵ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= (1)\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\right]$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

$= \left((\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta\right) - \sin^2\theta\cos^2\theta$ [∵ $a^2 + b^2 = (a + b)^2 - 2ab$]

$= (1)^2 - 3\sin^2\theta\cos^2\theta$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 - 3\sin^2\theta\cos^2\theta$

= R.H.S. (Proved)

11. Verify that $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ = 0$

Level-2 (understanding)

Ans:

L.H.S.

$= \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$

$= \tan(180^\circ + 45^\circ) \cot(360^\circ + 45^\circ) + \tan(360^\circ \times 2 + 45^\circ) \cot(360^\circ \times 2 - 45^\circ)$

$= \tan 45^\circ \cot 45^\circ + \tan 45^\circ (-\cot 45^\circ)$

$= \tan 45^\circ \cot 45^\circ - \tan 45^\circ \cot 45^\circ$

$= 0$

= R.H.S. (Verified)

12. Prove that $\tan 70^\circ = 2 \tan 50^\circ + \tan 20^\circ$ **Level-3 (Applying)**

Proof:

$\tan 70^\circ = \tan(50^\circ + 20^\circ)$

$\Rightarrow \tan 70^\circ = \frac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ . \tan 20^\circ}$

$\Rightarrow \tan 70^\circ (1 - \tan 50^\circ . \tan 20^\circ) = \tan 50^\circ + \tan 20^\circ$

$\Rightarrow \tan 70^\circ - \tan 70^\circ . \tan 50^\circ . \tan 20^\circ = \tan 50^\circ + \tan 20^\circ$

$\Rightarrow \tan 70^\circ - \tan(90^\circ - 20^\circ) . \tan 50^\circ . \tan 20^\circ = \tan 50^\circ + \tan 20^\circ$

$\Rightarrow \tan 70^\circ - \cot 20^\circ . \tan 50^\circ . \tan 20^\circ = \tan 50^\circ + \tan 20^\circ$

$\Rightarrow \tan 70^\circ - \tan 50^\circ = \tan 50^\circ + \tan 20^\circ$ $\quad [\because \cot 20^\circ . \tan 20^\circ = 1]$

$\Rightarrow \tan 70^\circ = 2 \tan 50^\circ + \tan 20^\circ$ (Proved)

13. Prove that $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ$

Proof: **Level-3 (Applying)**

L.H.S.

$= \sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ$

$= (\sin 50^\circ + \sin 10^\circ) + (\sin 40^\circ + \sin 20^\circ)$

$= 2\sin\left(\frac{50^\circ + 10^\circ}{2}\right)\cos\left(\frac{50^\circ - 10^\circ}{2}\right) + 2\sin\left(\frac{40^\circ + 20^\circ}{2}\right)\cos\left(\frac{40^\circ - 20^\circ}{2}\right)$

$= 2\sin 30^\circ \cos 20^\circ + 2\sin 30^\circ \cos 10^\circ$

$= 2\sin 30^\circ (\cos 20^\circ + \cos 10^\circ)$

$= 2\ \frac{1}{2}(\cos 20^\circ + \cos 10^\circ)$

$= \cos(90^\circ - 70^\circ) + \cos(90^\circ - 80^\circ)$
$= \sin 70^\circ + \sin 80^\circ$
$= \text{R. H. S.}$ (Proved)

14. Show that $\cos 20^\circ \; \cos 40^\circ \; \cos 60^\circ \; \cos 80^\circ = \frac{1}{16}$. Level-3 (Applying)

Proof:

L.H.S.
$= \cos 20^\circ \; \cos 40^\circ \; \cos 60^\circ \; \cos 80^\circ$
$= \frac{1}{2} \cos 20^\circ \; \cos 40^\circ \; \cos 80^\circ$ $\left[\because \cos 60^\circ = \frac{1}{2}\right]$
$= \frac{1}{2} \cos 20^\circ \; \frac{1}{2} (2 \cos 40^\circ \; \cos 80^\circ)$
$= \frac{1}{4} \cos 20^\circ \left\{ \cos(40^\circ + 80^\circ) + \cos(40^\circ - 80^\circ)\right\}$
$[\because 2 \cos A \cos B = \cos(A+B) + \cos(A-B)]$
$= \frac{1}{4} \cos 20^\circ \left(-\frac{1}{2} + \cos 40^\circ\right)$ $[\because \cos(-\theta) =$
$\cos\theta \;\; \& \;\; \cos 120^\circ = -\frac{1}{2}]$
$= -\frac{1}{8} \cos 20^\circ + \frac{1}{4} \cos 20^\circ \; \cos 40^\circ$
$= -\frac{1}{8} \cos 20^\circ + \frac{1}{4}\,\frac{1}{2} (2 \cos 20^\circ \; \cos 40^\circ)$
$= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} \left\{ \cos(20^\circ + 40^\circ) + \cos(20^\circ - 40^\circ)\right\}$
$= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} (\cos 60^\circ + \cos 20^\circ)$
$= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} \left(\frac{1}{2} + \cos 20^\circ\right)$
$= -\frac{1}{8} \cos 20^\circ + \frac{1}{16} + \frac{1}{8} \cos 20^\circ$
$= \frac{1}{16} = \text{R.H.S.}$ (Proved)

15. Show that $\frac{\sin 2A}{1 - \cos 2A} = \cot A$ and hence deduce the value of $\cot 15^\circ$

Level-2 (Understanding)

Proof:

$\text{L.H.S.} = \frac{\sin 2A}{1 - \cos 2A} = \frac{2 \sin A \cos A}{1 - (1 - 2\sin^2 A)} = \frac{2 \sin A \cos A}{2 \sin^2 A} = \frac{\cos A}{\sin A} = \cot A = \text{R.H.S.}$

$\therefore \cot A = \frac{\sin 2A}{1 - \cos 2A}$

Putting, $A = 15^\circ$, so that $2A = 30^\circ$

$\cot 15^\circ = \frac{\sin 30^\circ}{1 - \cos 30^\circ} = \frac{\frac{1}{2}}{1 - \frac{\sqrt{3}}{2}} = \frac{1}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$

16. Show that $\tan A + \tan B + \tan C = \tan A \,.\, \tan B \,.\, \tan C$, if A, B and C are the angles of a triangle.

Level-3(Applying)

Proof:

Given A, B and C are the angles of a triangle i.e.

$A+B+C=\pi$.

$\Rightarrow A+B=\pi-C$

$\Rightarrow \tan(A+B)=\tan(\pi-C)$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\ \tan B}=-\tan C$

$\Rightarrow \tan A+\tan B=-\tan C\,(1-\tan A\ \tan B)$

$\Rightarrow \tan A+\tan B+\tan C=\tan A\ \tan B\ \tan C$ (Proved)

17. prove that $\frac{\sin\left(\theta-\frac{\pi}{2}\right)}{\cos(\pi-\theta)}+\frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(\pi+\theta)}+\frac{\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)}{\sec(2\pi-\theta)}=3$ **Level-3 (Applying)**

Answer: $-\frac{\sin-\left(\frac{\Pi}{2}-\theta\right)}{\cos(\Pi-\theta)}+\frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(\Pi+\theta)}+\frac{\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)}{\sec(2\pi-\theta)}$

$=\frac{-\sin\left(\frac{\Pi}{2}-\theta\right)}{\cos(\Pi-\theta)}+\frac{\tan\left(\frac{\Pi}{2}-\theta\right)}{\cot(\pi+\theta)}+\frac{\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)}{\sec(2\Pi-\theta)}$

$=\frac{-\cos\theta}{-\cos\theta}+\frac{\cot\theta}{\cot\theta}+\frac{\sec\theta}{\sec\theta}=1+1+1=3$

18. Prove that $\frac{\sin 3\theta-\sin\theta}{\cos 3\theta+\cos\theta}=\tan\theta$ **Level-3 (Applying)**

Answer: $\frac{\sin 3\theta-\sin\theta}{\cos 3\theta+\cos\theta}$

$=\frac{2\cos\frac{3\theta+\theta}{2}\sin\frac{3\theta-\theta}{2}}{2\cos\frac{3\theta+\theta}{2}\cos\frac{3\theta-\theta}{2}}$

$=\frac{2\cos 2\theta\ \sin\theta}{2\cos 2\theta\ \cos\theta}=\tan\Theta$

19. Prove that sin10°sin30°sin50°sin70°=1/16 **Level-3 (Applying)**

Answer:- sin10°sin30°sin50°sin70

$=\frac{1}{2}\sin 10^0\sin 50^0\sin 70^0$

$=\frac{1}{2}\sin 10^0\sin(60^0-10^0)\sin(60^0+10^0)$

We know that sin(A+B)Sin(A-B)=$\sin^2$A-$\sin^2$B

$=\frac{1}{2}\sin 10^0(\sin^2 60^0-\sin^2 10^0)$

$=\frac{1}{2}\sin 10^0\left(\frac{3}{4}-\sin^2 10^0\right)$

$=\frac{1}{2}\frac{\sin 10^0(3-4\sin^2 10^0)}{4}=\frac{1}{8}(3\sin 10^0-4\sin^3 10^0)$

$=\frac{1}{8}\times\sin 3\times 10^0=\frac{1}{8}\sin 30^0$

$=\frac{1}{8}\times\frac{1}{2}=\frac{1}{16}$

20. Prove that cot 7½° = $\sqrt{6}+\sqrt{3}+\sqrt{2}+2$ **Level-3(Applying)**

Answer: We know $\cot\frac{\theta}{2}=\frac{1+\cos\theta}{\sin\theta}$

Here cot 7½°= $\cot\frac{15^\circ}{2}=\frac{1+\cos 15^\circ}{\sin 15^\circ}$

Now cos15° = cos(45°-30°) = cos45°.cos30° + sin45°.sin30°

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}$ $= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

sin15º = sin(45º-30º)=sin45º.cos30º - cos45º.sin30º

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

L.H.S

cot 7½º= $\cot\frac{15^o}{2} = \frac{1+\cos 15^o}{\sin 15^o}$ = $\frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\frac{2\sqrt{2}+\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

=

$$\frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1} = \frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{2\sqrt{6}+2\sqrt{2}+\sqrt{3}+\sqrt{3}+1+3}{3-1}$$

$= \frac{2\sqrt{6}+2\sqrt{3}+2\sqrt{2}+4}{2} = \sqrt{6}+\sqrt{3}+\sqrt{2}+2$ = R.H.S (proved)

21. If sin A = k sin B, Prove that tan½(A-B) = $\frac{k-1}{k+1}.\tan\frac{1}{2}(A+B)$

Level-2 (Understanding)

Answer: Given sinA=k sin B

$\Rightarrow \frac{\sin A}{\sin B} = \frac{k}{1}$

Using componendo & Dividendo rule, we get.

$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{k+1}{k-1}$

$\Rightarrow \frac{2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)} = \frac{k+1}{k-1}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right).\cot\left(\frac{A-B}{2}\right) = \frac{k+1}{k-1}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{k+1}{k-1}.\frac{1}{\cot\left(\frac{A-B}{2}\right)}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{k+1}{k-1}.\tan\left(\frac{A-B}{2}\right)$

$$\Rightarrow \quad \tan\left(\frac{A-B}{2}\right) = \frac{k-1}{k+1}.\tan\left(\frac{A+B}{2}\right) \quad \text{(proved)}$$

22. If A+B+C=π, then prove that sin2A+sin2B - sin2C = 4cosA. cosB.sinC

Level-3(Applying)

Answer : LHS = sin2A+sin2B-sin2C

$$= 2\sin\left(\frac{2A+2B}{2}\right).\cos\left(\frac{2A-2B}{2}\right) - \sin 2C.$$

= 2sin(A+B).cos(A-B)-sin2C

= 2 sin(π-C). cos(A-B)-sin2C.

= 2 sinC.cos(A-B)- 2sinC.cosC

= 2 sinC{cos(A-B) - cosC}

= 2 sinC{cos(A-B) - cos(-(A+B))}

= 2 sinC{cos(A-B)+cos(A+B)}

= 2 sinC.2cosA.cosB

= 4cosA. cosB.sinC = R.H.S (proved)

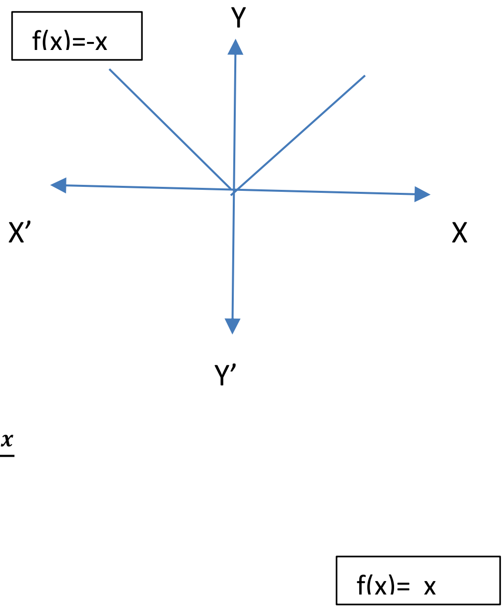
MATHEMATICS-1(1st Semester Diploma Engg:-2024-25)

UNIT-II

2 Marks Questions with Solutions — Taxonomy Level

1.Define Modulus Function and Draw the graph? **(L1- Remembering)**

Ans- The function defined by $f(x) = |x| = \begin{cases} x, when\ x \geq 0 \\ -x, when\ x < 0 \end{cases}$ $is\ called\ modulus\ function$



2.Evaluate $\lim_{x \to 0} \frac{sin3x}{x}$ **(L2- Understanding)**

Ans:-$\lim_{x \to 0} \frac{sin3x}{x}$

$= \lim_{x \to 0} \frac{sin3x}{3x}.\ 3$

$= \lim_{3x \to 0} \frac{sin3x}{3x}$ .3(since $3x \to 0\ as\ x \to 0$)

$= 3$. (since $\lim_{x \to 0} \frac{sinx}{x} = 1$)

3. if $f(x) = \tan x$,then prove that , $f(2x) = \frac{2f(x)}{1-[f(x)]^2}$ **(L2- Understanding)**

Ans- Given $f(x) = \tan x$

Then L.H.S= $f(2x) = \tan 2x$

$= \frac{2tanx}{1-\tan^2 x}$ (Formula of tan 2x)= $\frac{2f(x)}{1-[f(x)]^2}$ = R.H.S. . (since $\tan x = f(x)$)

4. Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1+cos2x}{1-cos2x}}$ **(L3- Applying)**

Ans-$y = \sqrt{\frac{1+cos2x}{1-c0s2x}}$

$= \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$ (since $1 + cos2x = 2\cos^2 x$ and $1 - cos2x = 2\sin^2 x$)

$= \frac{\cos x}{\sin x} = cot\, x.$

$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{1+cos2x}{1-c0s2x}}\right) = \frac{d}{dx}(cot\, x) =- cosec^2 x.$

5. Find derivative of the function $y = e^{5x+4}$ **(L2- Understanding)**

Ans:- Given $y = e^{5x+4}$

$$\frac{dy}{dx} = \frac{d}{dx}e^{5x+4}$$

$= e^{5x+4}.\frac{d}{dx}(5x + 4)$ (by using chain rule and $\frac{d}{dx}(e^x) = e^x$)

$= e^{5x+4}.5 = 5e^{5x+4}$.

6.Evaluate $\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}$ **(L3- Applying)**

Ans:- $\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}$

= $\lim_{x\to 0}\frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$

= $\lim_{x\to 0}\frac{x(\sqrt{x+1}+1)}{\sqrt{x+1}^2 - 1^2}$

= $\lim_{x\to 0}\frac{x(\sqrt{x+1}+1)}{x+1-1}$ = $\lim_{x\to 0}\frac{x(\sqrt{x+1}+1)}{x}$

= $\lim_{x\to 0}(\sqrt{x+1} + 1)$

= $\sqrt{0+1}$+1

= 1+1 = 2

7. Diffrentiate $y = x^3 + 3x^2 + 5$ w.r.t x and find $\frac{dy}{dx}$ at $x = 2$ **(L3- Applying)**

Ans-$y = x^3 + 3x^2 + 5$

$\Rightarrow \frac{dy}{dx} = 3x^2 + 6x + 0 \quad \frac{dy}{dx} = 3x^2 + 6x$

$\frac{dy}{dx}(at\ x = 2) = 3.(2)^2 + 6.2 = 12 + 12 = 24.$

$\therefore \ \frac{dy}{dx}$(at x=2)is 24.

8. Differentiate $\frac{1}{f(ax+b)}$ w.r.t. x. **(L3- Applying)**

Ans:-$\frac{d}{dx}\{\frac{1}{f(ax+b)}\}$

$= -\frac{1}{\{f(ax+b)\}^2} f'(ax+b) \frac{d}{dx}(ax+b)$

$= -\frac{a f'(ax+b)}{\{f(ax+b)\}^2}$

9. Find derivative of $y = \frac{\sin x}{x}$ **(L2- Understanding)**

Ans- given $y = \frac{\sin x}{x}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{sinx}{x}\right)$

$= \frac{\left(x.\frac{d}{dx}(sinx) - sinx.\frac{d}{dx}(x)\right)}{x^2}$ (by using quotient rule)

$= \frac{x.cosx - sinx.1}{x^2} = \frac{x\ cosx - sinx}{x^2}$ (Ans)

10. **Evaluate** $\lim_{x\to 0}\frac{sin5x}{sin7x}$ **(L2- Understanding)**

Solution: $\lim_{x\to 0}\frac{sin5x}{sin7x}$

$= \lim_{x\to 0}\frac{\frac{sin5x}{x}}{\frac{sin7x}{x}}$

$= \frac{\lim_{x\to 0}\frac{sin5x}{x}}{\lim_{x\to 0}\frac{sin7x}{x}}$

$= \frac{\lim_{x\to 0}\frac{sin5x}{5x}5}{\lim_{x\to 0}\frac{sin7x}{7x}7}$

$= \frac{5\lim_{x\to 0}\frac{sin5x}{5x}}{7\lim_{x\to 0}\frac{sin7x}{7x}}$

$= \frac{\mathbf{5.1}}{\mathbf{7.1}} = \frac{\mathbf{5}}{\mathbf{7}}$ (Ans)(As $x \to 0 \Rightarrow 5x \to 0$ and $7x \to 0$)

11. Evaluate $\lim_{x\to 0}\frac{3^x-1}{5^x-1}$ **(L2- Understanding)**

Solution: $\lim_{x\to 0}\frac{3^x-1}{5^x-1}$

$=\lim_{x\to 0}\frac{\frac{3^x-1}{x}}{\frac{5^x-1}{x}}$

$=\frac{\lim_{x\to 0}\frac{3^x-1}{x}}{\lim_{x\to 0}\frac{5^x-1}{x}}$

$=\frac{\ln 3}{\ln 5}$ (Ans)

12. Find $\lim_{x\to 0^+}[x]+10$ **(L2- Understanding)**

Solution: $\lim_{x\to 0^+}[x]+10$

= $\lim_{h\to 0}[0+h]+10$ (putting x = 0+h , As x->0 => h->0)

= 0 + 10

=10 (Ans)

13. Find the derivative of $tan\frac{\pi}{4}$ with respect to x. **(L2- Understanding)**

Solution: Derivative of$tan\frac{\pi}{4}$ with respect to x =$\frac{d}{dx}(tan\frac{\pi}{4})$

= 0 (As $tan\frac{\pi}{4}$ $is\ constant$)

14. If $y = lnx^2$ then find $\frac{dy}{dx}$. **(L2- Understanding)**

Solution: Given $y = lnx^2$=$2lnx$

$\frac{dy}{dx} = \frac{d}{dx}(2lnx)$

$= 2\frac{d}{dx}(lnx)$

$= 2\ \frac{1}{x}$

=$\frac{2}{x}$. (Ans)

15. Differentiate $\tan^{-1}x$ with respect to $\cot^{-1}x$. **(L3- Applying)**

Solution: Differentiating $\tan^{-1}x$ with respect to $\cot^{-1}x$

=$\frac{\frac{d}{dx}(\tan^{-1}x)}{\frac{d}{dx}(\cot^{-1}x)}$

=$\frac{\frac{1}{1+x^2}}{\frac{-1}{1+x^2}}$

=$\frac{1}{1+x^2}\frac{1+x^2}{-1}$

= -1 . (Ans)

16. If $y = lnsinx$ then find $\frac{dy}{dx}$. **(L2- Understanding)**

Solution: Given$y = lnsinx$

$$\frac{dy}{dx} = \frac{d}{dx}(lnsinx)$$
$$= \frac{\mathbf{1}}{\mathbf{sinx}}\frac{d}{dx}(sinx)$$
$$= \frac{\mathbf{1}}{\mathbf{sinx}}cosx \quad = \frac{\mathbf{cosx}}{\mathbf{sinx}}$$
$$= \cot x \text{ (Ans)}$$

17. What is the slope of the curve $y = tanx$ at x = $\frac{\pi}{4}$? **(L2- Understanding)**

Solution: Given$y = tanx$

Slope of the curve = $\frac{dy}{dx} = \frac{d}{dx}(tanx) = sec^2x$

At $= \frac{\pi}{4}, \left.\frac{dy}{dx}\right]_{x=\frac{\pi}{4}} = sec^2\frac{\pi}{4} = (\sqrt{2})^2 = 2$ (Ans)

18. Differentiate $lnsinx$ with respect to $tanx$. **(L3- Applying)**

Solution: Differentiating $lnsinx$ with respect to $tanx$

$$= \frac{\frac{d}{dx}(\boldsymbol{lnsinx})}{\frac{d}{dx}(\boldsymbol{tanx})}$$
$$= \frac{\frac{\mathbf{1}}{\mathbf{sinx}}\frac{d}{dx}(sinx)}{sec^2x}$$
$$= \frac{\frac{\mathbf{1}}{\mathbf{sinx}}cosx}{sec^2x}$$
$$= \frac{\mathbf{cotx}}{sec^2x}$$
$$= \text{cotx } \cos^2\text{x} \quad \text{(Ans)}$$

19. If $y = x^4 + e^x$, then find $\frac{dy}{dx}$ **(L2- Understanding)**

Solution: Given$y = x^4 + e^x$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 + e^x)$$
$$= \frac{d}{dx}(x^4) + \frac{d}{dx}(e^x)$$
$$= 4x^3 + e^x \text{(Ans)}$$

20. Evaluate $\lim_{x\to 0}\frac{\sin x^\circ}{x}$. **(L3- Applying)**

$Solution:- \lim_{x \to 0} \frac{\sin x^\circ}{x}$

$= \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{x} \quad (\because x^\circ = \frac{\pi x}{180} radian)$

$= \frac{\pi}{180} . \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}}$

$= \frac{\pi}{180} .1 \quad (\because \lim_{x \to 0} \frac{\sin x}{x} = 1)$

$= \frac{\pi}{180} \quad (Ans)$

21. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80,$ where n is a positive integer, find the value of $n.$

(L3- Applying)

Solution: $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$

$\Rightarrow \lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$

$\Rightarrow n.2^{n-1} = 80$

$\Rightarrow n2^n = 160$

$\Rightarrow n2^n = 5.2^5$

$\Rightarrow n = 5.$ (Ans)

22. If $y = \tan^{-1}(\sin^2 x)$, find $\frac{dy}{dx}$. **(L3- Applying)**

Solution: $y = \tan^{-1}(\sin^2 x)$

$\therefore \frac{dy}{dx} = \frac{1}{1 + \sin^4 x} . \frac{d}{dx}\left(\sin^2 x\right)$

$= \frac{1}{1 + \sin^4 x} 2 \sin x \cos x$

$= \frac{1}{1 + \sin^4 x} \sin 2x$. (Ans)

23. Find the derivative of $\sqrt{2x^2 + 3x + 5}$? **(L3- Applying)**

Solution: Let $y = \sqrt{2x^2 + 3x + 5}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{2x^2 + 3x + 5}} . \frac{d}{dx}\left(2x^2 + 3x + 5\right)$

= $\frac{1}{2\sqrt{2x^2+3x+5}}(4x+3)$. (Ans)

24. Differentiate $\sqrt{1+\sin 2x}$ w.r.t x ? **(L3- Applying)**

Solution: Let $y=\sqrt{1+\sin 2x}$

= $\sqrt{(\sin x+\cos x)^2}$

$=\sin x+\cos x$

$\therefore \frac{dy}{dx}=\cos x-\sin x$ (Ans)

25. Evaluate $\lim_{x\to 0}\left(\frac{3^{2x}-1}{x}\right)$? **(L2- Understanding)**

Solution: $\lim_{x\to 0}\left(\frac{3^{2x}-1}{x}\right)$

$=\lim_{x\to 0}\left(\frac{3^{2x}-1}{2x}\right)2=2\log 3$. (As $\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)=\log_e a$) (Ans)

26. **Evaluate** $\lim_{x\to 2}\frac{x^3-8}{x-2}$ **(L2- Understanding)**

Solution:$\lim_{x\to 2}\frac{x^3-8}{x-2}$

$=\lim_{x\to 2}\frac{x^3-2^3}{x-2}$

$=3.2^{3-1}$

$=3.2^2$

=3.4=12 . (Ans)

27. Evaluate $\lim_{x\to 0}\frac{cosx}{1+sinx}$ **(L2- Understanding)**

Solution:$\lim_{x\to 0}\frac{cosx}{1+sinx}$

$=\frac{cos0}{1+sin0}$

$=\frac{1}{1+0}=\frac{1}{1}=1$. (Ans)

28. Evaluate $\lim_{x\to 1}f(x)$ wheref(x) = $\begin{cases}-x & x<1\\ x+1 & x\geq 1\end{cases}$ **(L3- Applying)**

Ans:- L.H.L.= $\lim_{x\to 1^-}f(x)=\lim_{h\to 0}f(1-h)$ = $\lim_{h\to 0}$ – (1- h) = -(1-0) =-1

{ Putting x = 1-h $\Rightarrow x\to 1^-\Rightarrow h\to 0$ where h>0 and 1-h <0}

R.H.L.= $\lim_{x\to 1^+}f(x)=\lim_{h\to 0}f(1+h)$ = $\lim_{h\to 0}$ (1 + h)+1 = (1+0)+1 = 2

{ Putting x = 1+h $\Rightarrow x\to 1^+\Rightarrow h\to 0$ where h>0 and 1+h >0}

Hence L.H.L. ≠R.H.L.

$\therefore \lim_{x\to 1} f(x)$ does not exists.

29. Find $\frac{dy}{dx}$, if $y=\sin^{-1}2x$. **(L3- Applying)**

Solution: $y=\sin^{-1}2x$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-(2x)^2}}.\frac{d}{dx}(2x)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-4x^2}}.\frac{d}{dx}(2x)$$

$$\Rightarrow \frac{dy}{dx}=\frac{2}{\sqrt{1-4x^2}}$$. (Ans)

30. Differentiate sec(tanΘ) w.r.t. Θ. **(L3- Applying)**

Solution: Let y = sec(tanΘ)

$$\frac{dy}{d\theta}=\sec(\tan\theta)\tan(\tan\theta).\frac{d}{d\theta}(\tan\theta)$$

$$\Rightarrow \frac{dy}{d\theta}=\sec(\tan\theta)\tan(\tan\theta).\sec^2\theta$$ (Ans)

Extra question (2 Marks) to be added

1. Evaluate $\lim_{x\to\frac{1}{2}}\frac{Sin(2x-1)}{2x-1}$. **(L3- Applying)**

Solution: $\lim_{x\to\frac{1}{2}}\frac{Sin(2x-1)}{2x-1}\ \lim_{2x-1\to 0}\frac{Sin(2x-1)}{2x-1}=1\ [as\ x\to\frac{1}{2}\Rightarrow x-\frac{1}{2}\to 0 2x-1\to 0]$

2. Find the Derivative of $y=\sqrt{\frac{1-Cos2x}{1+Cos2x}}$ **(L3- Applying)**

Solution: $y=\sqrt{\frac{1-Cos2x}{1+Cos2x}}=\sqrt{\frac{2Sin^2x}{2Cos^2x}}=\sqrt{tan^2x}=tanx$

$$\frac{dy}{dx}=\frac{d(tanx)}{dx}=Sec^2x.$$

3. Evaluate $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ **(L3- Applying)**

Ans :- $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{2sin^2\frac{x}{2}}{x^2}$

$=\lim_{x\to 0}2.\frac{\sin\frac{x}{2}}{\frac{x}{2}2}.\frac{\sin\frac{x}{2}}{\frac{x}{2}2}=\frac{1}{2}\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}.\frac{\sin\frac{x}{2}}{\frac{x}{2}}=\frac{1}{2}\times 1\times 1=\frac{1}{2}$

4. Evaluate $\lim_{x\to 1}\frac{x^2-2x+1}{x^2-x}$ **(L3- Applying)**

Ans. $\lim_{x\to 1}\frac{x^2-2x+1}{x^2-x}$ = $\lim_{x-1}\frac{(x-1)^2}{x(x-1)}$

= $\lim_{x-1}\frac{x-1}{x}$

= $\frac{1-1}{1}$ = $\frac{0}{1}$ = 0

5. Differentiate $\tan^{-1} e^{2x}$ w.r.t x. **(L2- Understanding)**

Ans:- $\frac{dy}{dx} = \frac{1}{1+(e^{2x})^2}\frac{d}{dx}(e^{2x})$

$= \frac{2\,e^{2x}}{1+\,e^{4x}}$

6. Find f ' $(\sqrt{3})$ if f(x) = x $\tan^{-1} x$ **(L3- Applying)**

Ans. f(x) = $x\tan^{-1} x$

f ' (x) = $\frac{d(\text{x } \tan^{-1} x)}{dx}$ = $x\frac{1}{1+x^2}$ + 1. $\tan^{-1} x$

= $x\frac{1}{1+x^2} +\ \tan^{-1} x$

f ' $(\sqrt{3}) = \frac{\sqrt{3}}{1+(\sqrt{3})^2} + \tan^{-1}\sqrt{3}$

$= \frac{\sqrt{3}}{1+3} + \frac{\pi}{3}$

$= \frac{\sqrt{3}}{4} + \frac{\pi}{3}$

<u>**5 Marks Questions & Solutions**</u> <u>***Taxonomy Level***</u>

1. Estimate $\lim_{x\to 0}\left(1+\frac{3x}{4}\right)^{\frac{6}{x}}$ **(L3- Applying)**

Ans: $\lim_{x\to 0}\left(1+\frac{3x}{4}\right)^{\frac{6}{x}}$

$=\lim_{x\to 0}\left(1+\frac{3x}{4}\right)^{\frac{4}{3x}.\frac{6.3}{4}}$

$=\lim_{x\to 0}\left(1+\frac{3x}{4}\right)^{\frac{4}{3x}\frac{18}{4}}$

$=\lim_{\frac{3x}{4}\to 0}\left(1+\frac{3x}{4}\right)^{\frac{4}{3x}\frac{18}{4}}$

$=\lim_{y\to 0}(1+y)^{\frac{1}{y}\frac{18}{4}}$ (by taking $\frac{3x}{4}=y$)

$=e^{\frac{18}{4}}=e^{\frac{9}{2}}$ (since $\lim_{y\to 0}(1+y)^{\frac{1}{y}}=e$) (Ans)

2.If $y=x^y$, then find $\frac{dy}{dx}$ **(L3- Applying)**

Ans- Given that $y=x^y$

Taking logarithmic both side we get

$\log y=\log x^y$

$\Rightarrow logy=y\log x$

$\Rightarrow \frac{d}{dx}(\log y)=\frac{d}{dx}(y\,logx)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\log x\frac{d}{dx}(y)+y\frac{d}{dx}(\log x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\log x\,.\frac{dy}{dx}+y\,.\frac{1}{x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y}-logx\right)=y/x$

$\Rightarrow \frac{dy}{dx}\left(\frac{1-y\log x}{y}\right)=\frac{y}{x}$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}\left(\frac{y}{1-y\log x}\right)$

Therefore $\frac{dy}{dx}=\frac{y^2}{x(1-y\,logx)}$(Ans)

3. Evaluate $\lim_{x\to 5}\frac{\log_e x-\log_e 5}{x-5}$ **(L3- Applying)**

$=\lim_{x\to 5}\frac{\log_e x-\log_e 5}{x-5}$ (Put u = x – 5, when x-->5 then u --> 0)

$=\lim_{u\to 0}\frac{\log_e(u+5)-\log_e 5}{u}$

$=\lim_{u\to 0}\frac{\log_e\left(\frac{u+5}{5}\right)}{u}$

$= \lim\limits_{u \to 0} \frac{\log_e(\frac{u}{5}+1)}{u}$

$= \lim\limits_{u \to 0} \frac{\log_e(\frac{u}{5}+1)}{\frac{u}{5}.5}$

$= \frac{1}{5} \lim\limits_{u \to 0} \frac{\log_e(1+\frac{u}{5})}{\frac{u}{5}}$

$= \frac{1}{5} . 1 \;\; = \frac{1}{5}$ (Ans)

4. Evaluate $\lim\limits_{x \to 0} \frac{e^{sinx}-1}{x}$ **(L3- Applying)**

Solution: $\lim\limits_{x \to 0} \frac{e^{sinx}-1}{x}$

$= \lim\limits_{x \to 0} \frac{e^{sinx}-1}{x} \frac{sinx}{sinx}$

$= \lim\limits_{x \to 0} \frac{e^{sinx}-1}{sinx} \frac{sinx}{x}$

$= \lim\limits_{x \to 0} \frac{e^{sinx}-1}{sinx} \lim\limits_{x \to 0} \frac{sinx}{x}$

$= \lim\limits_{sinx \to 0} \frac{e^{sinx}-1}{sinx} \lim\limits_{x \to 0} \frac{sinx}{x}$ (As $x \to 0 \;\; => sinx \to 0$)

$= \ln e .1$

$=1.1 = 1$ (Ans)

5. Evaluate $\lim\limits_{x \to 0} \frac{3^x - 2^x}{4^x - 3^x}$ **(L3- Applying)**

Solution: $\lim\limits_{x \to 0} \frac{3^x - 2^x}{4^x - 3^x}$

$= \lim\limits_{x \to 0} \frac{\frac{3^x - 2^x}{x}}{\frac{4^x - 3^x}{x}}$

$= \frac{\lim\limits_{x \to 0} \frac{3^x - 2^x}{x}}{\lim\limits_{x \to 0} \frac{4^x - 3^x}{x}}$

$= \frac{\lim\limits_{x \to 0} \frac{(3^x - 1)-(2^x-1)}{x}}{\lim\limits_{x \to 0} \frac{(4^x - 1)-(3^x-1)}{x}}$

$= \frac{\lim\limits_{x \to 0} \frac{3^x - 1}{x} - \lim\limits_{x \to 0} \frac{2^x - 1}{x}}{\lim\limits_{x \to 0} \frac{4^x - 1}{x} - \lim\limits_{x \to 0} \frac{3^x - 1}{x}}$

$= \frac{\ln 3 - \ln 2}{\ln 4 - \ln 3}$

$= \frac{\ln \frac{3}{2}}{\ln \frac{4}{3}}$ (Ans)

6. Examine the existence of the limit of the function

$$f(x) = \begin{cases} \frac{|x-2|}{x-2}, & x \neq 2 \\ 0, & x = 2 \end{cases}$$ at x = 2 **(L3- Applying)**

Solution: Given $(x) = \begin{cases} \frac{|x-2|}{x-2}, & x \neq 2 \\ 0, & x = 2 \end{cases}$ at x = 2

L.H.L.= $\lim_{x \to 2^-} f(x)$

= $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$

=$\lim_{h \to 0} \frac{|2-h-2|}{2-h-2}$ (putting x = 2-h , As x->2 => h->0)

=$\lim_{h \to 0} \frac{|-h|}{-h}$

= $\lim_{h \to 0} \frac{h}{-h}$

=$\lim_{h \to 0} -1$

= -1

R.H.L. = $\lim_{x \to 2^+} f(x)$

= $\lim_{x \to 2^+} \frac{|x-2|}{x-2}$

=$\lim_{h \to 0} \frac{|2+h-2|}{2+h-2}$ (putting x = 2+h , As x->2 => h->0)

=$\lim_{h \to 0} \frac{|h|}{h}$

= $\lim_{h \to 0} \frac{h}{h}$

=$\lim_{h \to 0} 1$

= 1

∴ L.H.L ≠ R.H.L

$Hence \ \lim_{x \to 2} \frac{|x-2|}{x-2}$ does not exist.

7. Find the derivative of sinx by using definition or first principle. **(L3- Applying)**

Solution: Given y = f(x) = sinx

As x changes to x+h

=> f(x) = sinx changes to f(x+h) = sin(x+h)

$$\therefore \ \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

=$\lim_{h \to 0} \frac{sin(x+h) - sinx}{h}$

=$\lim_{h \to 0} \frac{2\,cos\left(\frac{x+h+x}{2}\right)\ \sin\left(\frac{x+h-x}{2}\right)}{h}$

=$\lim_{h \to 0} \frac{2\,cos\left(\frac{2x+h}{2}\right)\ \sin\left(\frac{h}{2}\right)}{h}$

=$\lim_{h \to 0} \frac{2\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

=$\lim_{h \to 0} \cos\left(x + \frac{h}{2}\right) \lim_{h \to 0} \frac{2\sin\left(\frac{h}{2}\right)}{h}$

= $\cos\left(x + \frac{0}{2}\right) \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

$=\cos x \ . 1$

$= \cos x$

$\therefore \frac{d}{dx}(sinx) = cosx$ (Ans.)

8. Differentiate $x^3 + y^3 = 3axy$ with respect to x.

Solution: Given $x^3 + y^3 = 3axy$ ------------(1) **(L3- Applying)**

Differentiating w.r.t x on both sides of (1) we have

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3axy)$$

$=>\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = 3a\frac{d}{dx}(xy)$

$=>3x^2 + 3y^2\frac{dy}{dx}$ =3a ($x\frac{dy}{dx} + y\frac{dx}{dx}$)

$=>3(x^2 + y^2\frac{dy}{dx}) = 3a(x\frac{dy}{dx} + y.1)$

$=> 3(x^2 + y^2\frac{dy}{dx}) = 3a(x\frac{dy}{dx} + y)$

$=>x^2 + y^2\frac{dy}{dx} = a(x\frac{dy}{dx} + y)$

$=>x^2 + y^2\frac{dy}{dx} = ax\frac{dy}{dx} + ay$

$=>y^2\frac{dy}{dx} - ax\frac{dy}{dx} = ay - x^2$

$=>(y^2 - ax)\frac{dy}{dx} = ay - x^2$

$=>\frac{dy}{dx} = \frac{ay-x^2}{y^2-ax}$ (Ans.)

9. Find $\frac{dy}{dx}$ if $y = 5^{\sin x^2}$. **(L3- Applying)**

Solution: Given that, $y = 5^{\sin x^2}$

Differentiating both sides w.r.t. x.

$$\frac{dy}{dx} = 5^{\sin x^2} \ln 5 \frac{d}{dx}(\sin x^2)$$

$= 5^{\sin x^2} \ln 5 \cos x^2 . \frac{d}{dx}(x^2)$ (As $\frac{d(a^x)}{dx} = a^x$ lna)

$= 5^{\sin x^2} \ln 5 \cos x^2 . 2x$

$= 2x5^{\sin x^2} \ln 5 \cos x^2 \quad (Ans)$

10. Differentiate $(\log x)^{\tan x}$. **(L3- Applying)**

Solution: Let $y = (\log x)^{\tan x}$

Taking logarithm both sides

$\ln y = \ln (\log x)^{\tan x}$.

$= \tan x . \ln(\log x)$

Now differentiating both sides w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = \ln(\log x).\sec^2 x + \tan x.\frac{1}{x\log x} \quad \left(\text{As } \frac{d(logx)}{dx} = \frac{1}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\ln(\log x).\sec^2 x + \tan x.\frac{1}{x\log x}\right\}$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\tan x}\left\{\ln(\log x).\sec^2 x + \tan x.\frac{1}{x\log x}\right\} \quad \text{(Ans)}$$

Extra question (5 Marks) to be added

1.Evaluate $\lim_{x\to 0}\frac{5xCosx-2Sinx}{7x+5Sinx}$ **(L3- Applying)**

Solution: $\lim_{x\to 0}\frac{5xCosx-2Sinx}{7x+5Sinx}$

$= \lim_{x\to 0}\frac{x[5Cosx-2\frac{Sinx}{x}]}{x[7+5\frac{Sinx}{x}]}$

$= \frac{5\lim_{x\to 0}Cosx-2\lim_{x\to 0}\frac{Sinx}{x}}{7+5\lim_{x\to 0}\frac{Sinx}{x}}$

$= \frac{5X1-1X2}{7+5X1} = \frac{1}{4}$

2. Find the Derivative of $\cos^{-1}\frac{1-x^2}{1+x^2}$ with respect to x . **(L3- Applying)**

Solution: Let Y =$\cos^{-1}\frac{1-x^2}{1+x^2}$

To find the derivative , put $x = tan\theta$ $so\ that\ \theta = tan^{-1}x$

$$Y = \cos^{-1}\left(\frac{1-tan^2\theta}{1+tan^2\theta}\right) = \cos^{-1}Cos2\theta = 2\theta$$

$\frac{dy}{dx} = \frac{d(2\theta)}{dx} = \frac{d(2\theta)}{d\theta}X\frac{d\theta}{dx} = 2X\frac{d(\tan^{-1}x)}{dx} = \frac{2}{1+x^2}$.

3. Evaluate $\lim_{x\to 0}\frac{e^{tanx}-1}{x}$. **(L3- Applying)**

Solution: $\lim_{x\to 0}\frac{e^{tanx}-1}{x} = \lim_{x\to 0}\frac{e^{tanx}-1}{tanx}X\frac{tanx}{x} = \lim_{x\to 0}\frac{e^{tanx}-1}{tanx}X\lim_{x\to 0}\frac{tanx}{x}$

$$= \lim_{tanx\to 0} \frac{e^{tanx}-1}{tanx} X \lim_{x\to 0} \frac{tanx}{x} = 1X1 = 1 \quad [as\ x \to 0\ \tan x \to 0]$$

4. Evaluate $\lim_{x\to\infty} \left(1+\frac{2}{3x}\right)^{5x}$ **(L3- Applying)**

Solution: $\lim_{x\to\infty} \left(1+\frac{2}{3x}\right)^{\frac{3x}{2}X\frac{10}{3}}$

$$= \left\{\lim_{\frac{2}{3x}\to 0} \left(1+\frac{2}{3x}\right)^{\frac{1}{\frac{2}{3x}}}\right\}^{\frac{10}{3}}$$

$$= e^{\frac{10}{3}} \qquad \left[as\ x \to \infty \frac{1}{x} \to 0 \frac{2}{3x} \to 0\right]$$

5. Find the value of a if $\lim_{x\to 1} \frac{5^x-5}{(x-1)\log_e a} = 5$ **(L3- Applying)**

Solution: $\lim_{x\to 1} \frac{5(5^{x-1}-1)}{(x-1)\log_e a} = 5$

$\lim_{y\to 0} \frac{5(5^y-1)}{y\log_e a} = 5$ putting y= x-1 when $x \to 1\ \ y \to 0$

$\Rightarrow \frac{5}{\log_e a} \lim_{y\to 0} \frac{(5^y-1)}{y} = 5$

$\Rightarrow \frac{\log_e 5}{\log_e a} = 1 \Rightarrow \log_e 5 = \log_e a \Rightarrow$ a = 5 .

6. Find the Derivative of $y = \log[\log(logx)]$ **(L3- Applying)**

Solution: $y = \log[\log(logx)]$

$$\frac{dy}{dx} = \frac{d}{dx}\log[\log(logx)] = \frac{1}{\log(logx)} \frac{d\{\log(logx)\}}{dx} = \frac{1}{\log(logx)} \frac{1}{logx} \frac{d(logx)}{dx}$$

$$= \frac{1}{\log(logx)} \frac{1}{logx} \frac{1}{x} = \frac{1}{xlogx.\log(logx)}$$

7. Evaluate $\lim_{x\to 0} \left(\frac{\tan x - \sin x}{sin^3 x}\right)$ **(L3- Applying)**

$$\lim_{x\to 0} \left(\frac{\tan x - \sin x}{sin^3 x}\right) = \lim_{x\to 0} \left(\frac{\frac{\sin x}{\cos x} - \sin x}{sin^3 x}\right)$$

$$= \lim_{x\to 0} \left(\frac{\sin x - \sin x \cos x}{sin^3 x \cos x}\right) = \lim_{x\to 0} \left(\frac{\sin x(1-cosx)}{sin^3 x \cos x}\right) = \lim_{x\to 0} \left(\frac{1-\cos x}{sin^2 x \cos x}\right)$$

$$= \lim_{x\to 0}\left(\frac{1-cosx}{(1-cos^2x)cosx}\right) = \lim_{x\to 0}\left(\frac{(1-cosx)}{(1-cosx)(1+cosx)cosx}\right)$$

$$= \lim_{x\to 0}\left(\frac{1}{(1+\cos x)\cos x}\right) = \frac{1}{(1+cos0)cos0} = \frac{1}{(1+1).1} = \frac{1}{2}$$

8. Evaluate $\lim_{x\to 1}\frac{x^2-4x+3}{x^2-6x+5}$ **(L3- Applying)**

Ans.

$$\lim_{x\to 1}\frac{x^2-3x-x+3}{x^2-5x-x+5} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 1}\frac{x(x-3)-1(x-3)}{x(x-5)-1(x-5)}$$

$$= \lim_{x\to 1}\frac{(x-3)(x-1)}{(x-5)(x-1)}$$

$$= \lim_{x\to 1}\frac{x-3}{x-5} = \frac{1-3}{1-5} = \frac{-2}{-4} = \frac{1}{2}$$

9. Evaluate $\lim_{x\to 0}\frac{3^x+3^{-x}-2}{x^2}$ **(L3- Applying)**

Ans.

$$\lim_{x\to 0}\frac{3^x+3^{-x}-2}{x^2} = \lim_{x\to 0}\frac{3^x+\frac{1}{3^x}-2}{x^2}$$

$$= \lim_{x\to 0}\frac{3^{2x}+1-2\,3^x}{3^x x^2} = \lim_{x\to 0}\frac{(3^x)^2-2.3^x.1+1^2}{3^x x^2}$$

$$= \lim_{x\to 0}\frac{(3^x-1)^2}{3^x x^2} = \lim_{x\to 0}\frac{1}{3^x}\left(\frac{3^x-1}{x}\right)^2$$

$$= \frac{1}{3^\circ}(\log_e 3)^2$$

$$= (\ln 3)^2$$

10. **Find $\frac{dy}{dx}$ where** y = $\sqrt{\sin\sqrt{x}}$ **(L3- Applying)**

Ans:- y = $\sqrt{\sin\sqrt{x}}$

Here y = $\sqrt{u}$, u = $\sin v$, v = $\sqrt{x}$

So, $\frac{dy}{du} = \frac{1}{2\sqrt{u}}$, $\frac{du}{dv}$ = cos v, $\frac{dv}{dx} = \frac{1}{2\sqrt{x}}$

Therefore, $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dv} . \frac{dv}{dx}$

$= \frac{1}{2\sqrt{u}}$. cos v. $\frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{\sin v}}$. cos $\sqrt{x}$. $\frac{1}{2\sqrt{x}}$

$= \frac{cos\sqrt{x}}{4\sqrt{sin\sqrt{x}}\sqrt{x}}$

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11. If y = $sec^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)$ then find $\frac{dy}{dx}$ **(L3- Applying)**

Ans.

y $= \sec^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)$ Put x = a tan θ

$= \sec^{-1}\left(\frac{\sqrt{a^2+a^2tan^2\theta}}{a}\right)$ $= \sec^{-1}\left(\frac{\sqrt{a^2(1+tan^2\theta)}}{a}\right)$

$= \sec^{-1}\left(\frac{\sqrt{a^2sec^2\theta}}{a}\right)$ $= \sec^{-1}\left(\frac{a\sec\theta}{a}\right)$

$= \sec^{-1}(\sec\theta) = \theta$ $= \tan^{-1}\left(\frac{x}{a}\right)$

Now $\frac{dy}{dx} = \frac{d}{dx}\{\tan^{-1}\left(\frac{x}{a}\right)\} = \frac{1}{1+\left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$

$= \frac{1}{(1+\frac{x^2}{a^2})}\left(\frac{1}{a}\right) = \frac{1}{a}\frac{1}{\frac{a^2+x^2}{a^2}}$

$= \frac{a^2}{a(x^2+a^2)} = \frac{a}{x^2+a^2}$

12. Differentiate $(\sin x)^{\ln x}$ w.r.t x **(L3- Applying)**

Ans.

y $= (\sin x)^{\ln x}$

Then log y = log $(\sin x)^{\ln x}$ = In x log (sin x)

Differentiating w.r.t x ,

$\frac{1}{y}\frac{dy}{dx} = lnx\frac{1}{\sin x}cosx + \frac{1}{x}$log(sin x)

$\Rightarrow \frac{dy}{dx} = y\,[\ \ \ln x \cot x + \frac{\log(\sin x)}{x}]$

$\therefore \frac{dy}{dx} = (\sin x)^{\ln x}\,[\ \ \ln \cot x + \frac{\log(\sin x)}{x}]$.

MATHEMATICS-1(1st Semester Diploma Engg:-2024-25)
UNIT-III

02 Mark Questions & Solutions **Taxonomy Level**

1. Find the value of $(-\mathbf{i})^{\mathbf{4n+2}}$. Level-2(Understanding)

Solution:
$(-i)^{4n+2} = (-i)^{4n} \cdot (-i)^2 = ((-i)^4)^n \cdot (-i)^2 = i^{4^n} \cdot -1 = 1^n \cdot -1 = -1$

2. **What is the value of $\arg(\omega) + \arg(\omega^2)$?** Level-2(Understanding)

Solution:

Since $\arg(z_1) + \arg(z_2) = \arg(z_1 . z_2)$.

$\Rightarrow \arg(\omega) + \arg(\omega^2) = \arg(\omega \times \omega^2) = \arg(\omega^3) = \arg(1) = 0.$

Therefore, $\arg(\omega) + \arg(\omega^2) = 0$

3. **Find the conjugate of $\frac{1}{3+4i}$.** Level-2(Understanding)

Solution:
$$\text{Let } z = \frac{1}{3+4i} = \frac{1(3-4i)}{(3+4i)(3-4i)} = \frac{3-4i}{3^2-(4i)^2} = \frac{3-4i}{9-16(i)^2}$$
$$= \frac{3-4i}{9-16(-1)} = \frac{3-4i}{9+16} = \frac{3-4i}{25} = \frac{3}{25} - \frac{4}{25}i$$

$$\text{So, } \bar{z} = \frac{3}{25} + \frac{4}{25}i$$

$$\text{Thus, conjugate of } \frac{1}{3+4i} = \frac{3}{25} + \frac{4}{25}i$$

4. Find the multiplicative inverse of $\mathbf{4 - 3i}$. Level-2(Understanding)

Solution:

$\bar{z} = 4 + 3i$ & $|z| = \sqrt{4^2 + 3^2} = \sqrt{25}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

Thus, the multiplicative inverse of $4 - 3i$ is $\frac{4}{25} + \frac{3}{25}i$

5. **Express in the form of $a + ib$, if $z = \frac{(1+i)^2}{3-i}$.** Level-2(Understanding)

Solution:

Since $z = \frac{(1+i)^2}{3-i} = \frac{1^2 + i^2 + 2.1.i}{3-i} = \frac{1-1+2i}{3-i} = \frac{2i}{3-i} = \frac{2i(3+i)}{(3-i)(3+i)} = \frac{6i+2i^2}{3^2-i^2} = \frac{6i+2(-1)}{9—1}$
$= \frac{6i-2}{9+1}$

$= \frac{-2+6i}{10} = \frac{-2}{10} + \frac{6}{10}i = \frac{-1}{5} + \frac{3}{5}i$

$\therefore a = \frac{-1}{5}$ and $b = \frac{3.}{5}$

Thus, a+ib, form of $z = \frac{(1+i)^2}{3-i}$ is $\frac{-1}{5} + \frac{3}{5}i$

6. **Express $-1 + \sqrt{3i}$ in polar form.** Level-2(Understanding)

Solution:

$\mathbb{Z} = -1 + \sqrt{3}i = |\mathbb{Z}|e^{i\theta}$ (where $|\mathbb{Z}| =$ modulus of the complex number and θ $=$ amplitude of the complex number)

$|\mathbb{Z}| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1` + 3} = \sqrt{4} = 2$

$\theta = \pi - \tan^{-1}\frac{\sqrt{3}}{1} = \pi - \tan^{-1}\sqrt{3} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore \mathbb{Z} = 2e^{\frac{i2\pi}{3}}$

7. **Find the value of x and y if $(x-2y) + 3yi = 4 - 6i$.** Level-2(Understanding)

Solution:

comparing the real and imaginary parts we get ,

$x - 2y = 4, 3y = -6$

$=> 3y = -6, \quad => y = \frac{-6}{3} = -2$

now , $x - 2(-2) = 4$

$=> x + 4 = 4$

$=> x = 4 - 4 = 0$

Thus, x=0, y =-2

8. **If w is the cube root of unity find the value of $(1+w)^5$** Level-2(Understanding)

Solution:

$(1+w)^5 = (-w^2)^5$ (since $1 + w + w^2 = 0 \therefore (1+w) = -w^2$)

$\therefore (1+w)^5 = -w^{10} = -w^9.w = -(w^3)^3.w = -(1)^3.w = -1.w = -w$ (since$w^3 = 1$)

9. **Find the square root of $2i$.** Level-2(Understanding)

Solution:

$\sqrt{2i} = \sqrt{1-1+2i} = \sqrt{(1)^2+(i)^2+2i} = \sqrt{(1+i)^2} = \pm(1+i)$

10. **Find the value of $\mathbf{i^{17}+i^{20}-i^{13}}$.** Level-2(Understanding)

Solution:

$i^{17}+i^{20}-i^{13}$

$= i^{16}.i + \left(i^4\right)^5 - i^{12}.i$

$= (i^4)^4.i + (i^4)^5 - (i^4)^3.i = 1^4.i + 1^5 - 1^3.i$ (since $i^4 = 1$)

$= i + 1 - i$

$= 1$

11. **Find the multiplicative inverse of $\mathbf{2+3i}$.** Level-2(Understanding)

Solution:

Let $z = 2+3i$

Multiplicative inverse of $z = \dfrac{1}{z} = \dfrac{1}{2+3i} = \dfrac{1(2-3i)}{(2+3i)(2-3i)} = \dfrac{(2-3i)}{2^2-(3i)^2}$

$= \dfrac{(2-3i)}{4-9(-1)} = \dfrac{(2-3i)}{4+9} = \dfrac{(2-3i)}{13} = \dfrac{2}{13} - \dfrac{3}{13}i$

12. **Find the modulus and amplitude of $\mathbf{\frac{1}{1-i}}$.** Level-2(Understanding)

Solution:

Let $z = \dfrac{1}{1-i} = \dfrac{1(1+i)}{(1-i)(1+i)} = \dfrac{1+i}{1^2-i^2} = \dfrac{1+i}{1+1} = \dfrac{1+i}{2} = \dfrac{1}{2} + \dfrac{1}{2}i$

$\therefore z = \dfrac{1}{2} + \dfrac{1}{2}i$, Here $a = \dfrac{1}{2}$ and $b = \dfrac{1}{2}$

$|z| = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2} = \sqrt{\dfrac{1}{4}+\dfrac{1}{4}} = \sqrt{\dfrac{2}{4}} = \sqrt{\dfrac{1}{2}} = \dfrac{1}{\sqrt{2}}$

Amplitude $(\theta) = \tan^{-1}\left(\dfrac{b}{a}\right) = \tan^{-1}\left(\dfrac{\frac{1}{2}}{\frac{1}{2}}\right) = \tan^{-1}(1) = \dfrac{\pi}{4.}$

13. **Prove that $\mathbf{P(n,n) = P(n,n-1)}$.** Level-2(Understanding)

Proof:

$L.H.S. = P(n,n) = \dfrac{n!}{(n-n)!} = \dfrac{n!}{(0)!} = \dfrac{n!}{1} = n!$

$$\text{R.H.S.} = P(n, n-1) = \frac{n!}{(n-(n-1))!} = \frac{n!}{(n-n+1)!} = \frac{n!}{(1)!} = n!$$

$\therefore$ L.H.S. = R.H.S. (Proved)

14. **If $P(n, r) = 1680$, $C(n, r) = 70$, then find n and r.** Level-2(Understanding)

Solution:

Given that $P(n, r) = 1680, C(n, r) = 70$

$$\frac{P(n, r)}{C(n, r)} = \frac{1680}{70}$$

$$\Rightarrow \frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!\,(n-r)!}} = 24$$

$$\Rightarrow \frac{n!}{(n-r)!} \times \frac{r!\,(n-r)!}{n!} = 24$$

$\Rightarrow r! = 24 = 4!$

$\Rightarrow r = 4.$

Since $P(n, r) = 1680$

$P(n, 4) = 1680$

$$\Rightarrow \frac{n!}{(n-4)!} = 1680$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 8 \times 7 \times 6 \times 5$$

$\Rightarrow n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$

$\Rightarrow n = 8$

15. If **$C(20, r+6) = C(20, r+2)$, then find r.** Level-2(Understanding)

Solution:

Given $C(20, r+6) = C(20, r+2)$
$\Rightarrow r + 6 + r + 2 = 20$
$\Rightarrow 2r + 8 = 20$
$\Rightarrow 2r = 12$
$\Rightarrow r = 6.$

16. **Compute $P(n, r)$ and $C(n, r)$, if $n = 10$ and $r = 3$.** Level-2(Understanding)

Solution:

$$P(n,r) = P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10\times 9\times 8\times 7!}{7!} = 720.$$

$$C(n,r) = C(10,3) = \frac{10!}{3!\,(10-3)!} = \frac{10!}{3!\,7!} = \frac{10\times 9\times 8\times 7!}{(3\times 2\times 1)7!} = 120.$$

17. **Find the10th term of** $\left(2x^2 + \frac{1}{x}\right)^{12}$. Level-2(Understanding)

Solution:

$$t_{10} = t_{9+1} = C(12,9)\left(2x^2\right)^{12-9}\left(\frac{1}{x}\right)^9 = C(12,9)\left(2x^2\right)^3\frac{1}{x^9} = \frac{12!}{9!\,(12-9)!}2^3x^6\frac{1}{x^9} = \frac{12!}{9!\,3!}2^3\frac{1}{x^3}$$
$$= \frac{12\times 11\times 10\times 9!}{9!\,(3\times 2\times 1)}2^3\frac{1}{x^3} = \frac{1760}{x^3}$$

18. **Find the middle term of** $\left(2x^2 - \frac{1}{x}\right)^7$. Level-2(Understanding)

Solution:

Number of terms in this expansion is 8. Hence there are two middle terms

i. e. 4th term and 5th term.

$$4\text{th term} = t_4 = t_{3+1} = (-1)^3C(7,3)\left(2x^2\right)^{7-3}\left(\frac{1}{x}\right)^3 = (-1)\frac{7!}{3!\,(7-3)!}\left(2x^2\right)^4\frac{1}{x^3}$$
$$= (-1)\frac{7!}{3!\,4!}2^4x^8\frac{1}{x^3} = (-1)\frac{7\times 6\times 5\times 4!}{(3\times 2\times 1)4!}2^4x^5 =- 560x^5.$$

$$5\text{th term} = t_5 = t_{4+1} = (-1)^4C(7,4)\left(2x^2\right)^{7-4}\left(\frac{1}{x}\right)^4 = \frac{7!}{4!\,(7-4)!}\left(2x^2\right)^3\frac{1}{x^4} = \frac{7!}{3!\,4!}2^3x^6\frac{1}{x^4}$$
$$= \frac{7\times 6\times 5\times 4!}{(3\times 2\times 1)4!}2^3x^2 = 280x^2.$$

19. **Find the number of term in the expansion** $\left(x^2 - 2 + \frac{1}{x^2}\right)^6$. Level-2(Understanding)

Solution:

$$\left(x^2 - 2 + \frac{1}{x^2}\right)^6 = \left((x)^2 - 2\times x\times\frac{1}{x} + \left(\frac{1}{x}\right)^2\right)^6 = \left[\left(x - \frac{1}{x}\right)^2\right]^6 = \left(x - \frac{1}{x}\right)^{12}$$

∴ Total number of term in this expansion is 13.

20. **In the expansion of** $(1+x)^n$, **prove that** $C_0 + C_1 + C_2 + C_3 + \cdots\cdots + C_n = 2^n$.
Level-3(Applying)

Solution:

By Binomial theorem, we have

$$(1+x)^n = C_0 + C_1\,x + C_2x^2 + C_3x^3 + \cdots\cdots + C_nx^n \cdots\cdots\cdots \text{(Eq1)}$$

Putting $x = 1$, in (Eq1), we get

$(1+1)^n = C_0 + C_1 + C_2 + C_3 + \cdots\cdots + C_n$

$=> C_0 + C_1 + C_2 + C_3 + \cdots\cdots + C_n = 2^n.$

21. If $\mathbf{C(18, r) = C(18, r+2)}$, **then find** $\mathbf{C(r, 5)}$. Level-2(Understanding)

Solution:

$C(18, r) = C(18, r+2)$

$=> r + (r+2) = 18$

$=> 2r + 2 = 18$

$=> 2r = 18$-2=16

$=> r = 8$

$$\therefore C(r, 5) = C(8,5) = \frac{8!}{5!\ (8-5)!} = \frac{8 \times 7 \times 6 \times 5!}{5!\ 3!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

22. Find n, if $\mathbf{P(n, 4) : P(n-1, 3) = 9 : 1}$. Level-3(Applying)

Solution:

Given $P(n, 4) : P(n-1, 3) = 9 : 1$

$$=> \frac{P(n, 4)}{P(n-1, 3)} = \frac{9}{1}$$

$$=> \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)!}} = 9$$

$$=> \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-4)!}} = 9$$

$$=> \frac{n(n-1)!}{(n-1)!} = 9$$

$=>$ n=9.

23. If $\mathbf{n}$, $\mathbf{r}$ are positive integers, such that $\mathbf{1 \le r \le n}$, then show that $\frac{\mathbf{C(n.r)}}{\mathbf{C(n, r-1)}} = \frac{\mathbf{n-r+1}}{\mathbf{r}}$

Level-3(Applying)

Solution:

<u>L. H. S.</u>

$$\frac{C(n.r)}{C(n, r-1)} = \frac{\frac{n!}{r!\ (n-r)!}}{\frac{n!}{(r-1)!\,(n-(r-1))!}} = \frac{n!}{r!\ (n-r)!} \times \frac{(r-1)!\,(n-(r-1))!}{n!}$$

$$= \frac{(r-1)!\,(n-r+1)!}{r \times (r-1)!\ (n-r)!} = \frac{(n-r+1)(n-r)!}{r\,(n-r)!}$$

$= \frac{(n-r+1)}{r} = \text{RHS(proved)}$

24. Show that $\mathbf{P(n,r) = P(n-1,r) + rP(n-1,r-1)}$. Level-3(Applying)

Solution:

R.H.S.

$$P(n-1,r) + rP(n-1,r-1) = \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{(n-1-(r-1))!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-1-r+1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[1 + \frac{r}{(n-r)}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[\frac{n-r+r}{(n-r)}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[\frac{n}{(n-r)}\right] = \frac{n!}{(n-r)!}$$

$$= P(n,r) = LHS(Proved)$$

25. Find the middle terms in the expansion of $\left(\frac{\boldsymbol{a}}{\boldsymbol{x}} + \frac{\boldsymbol{x}}{\boldsymbol{a}}\right)^{\mathbf{10}}$. Level-2(Understanding)

Solution:

The Required middle term $= t_{\frac{10}{2}+1}$

$$= t_6 = t_{5+1} = C(10,5)\left(\frac{a}{x}\right)^{10-5}\left(\frac{x}{a}\right)^5 = \frac{10!}{5!\,(10-5)!}\left(\frac{a}{x}\right)^5\left(\frac{x}{a}\right)^5$$

$$= \frac{10\times9\times8\times7\times6\times5!}{5!5\times4\times3\times2\times1}\frac{a^5}{x^5}\frac{x^5}{a^5} = 252.$$

26. Using Binomial Theorem, Find the value of $(\mathbf{99})^{\mathbf{4}}$. Level-3(Applying)

Solution:

$$(99)^4 = (100-1)^4$$

$$= C(4,0)100^4 - C(4,1)100^3 + C(4,2)100^2 - C(4,3)100 + C(4,4)$$

$$= 100^4 - 4\times100^3 + 6\times100^2 - 4\times100 + 1$$

$(\because C(4,0) = C(4,4) = 1, C(4,1) = C(4,3) = 4 \text{ and } C(4,2) = 6)$

$= 100000000 - 4000000 + 60000 - 400 + 1 = 96059601.$

27. Find the co-efficient of $\boldsymbol{x^{32}}$ in the expansion of $\left(\boldsymbol{x^4} - \frac{\mathbf{1}}{\boldsymbol{x^3}}\right)^{\mathbf{15}}$. Level-2(Understanding)

Solution:

Let (r+1)th term contains coefficient of x^{32}.

$$t_{r+1} = (-1)^r C(15,r)\left(x^4\right)^{15-r}\left(\frac{1}{x^3}\right)^r$$

$$= (-1)^r C(15,r)x^{60-4r}\frac{1}{x^{3r}}$$

$$= (-1)^r C(15,r)x^{60-4r-3r} = (-1)^r C(15,r)x^{60-7r}$$

$\therefore 60 - 7r = 32$

$=> 7r = 60 - 32 = 28.$

$=> r = 4$

$\therefore\ t_{4+1} = t_5 = (-1)^4 C(15,4) x^{60-7\times 4} = C(15,4)x^{32}$

$\therefore$The co-efficient of x^{32} is $C(15,4)$.

28. Find the 5th term in the expansion of $\left(\boldsymbol{6x} - \frac{a^3}{x}\right)^{10}$. Level-2(Understanding)

Solution:

$$t_5 = t_{4+1} = (-1)^4 C(10,4)(6x)^{10-4}\left(\frac{a^3}{x}\right)^4$$

$$= C(10,4)(6x)^6 \frac{(a^3)^4}{x^4}$$

$$= C(10,4)6^6x^6\frac{a^{12}}{x^4} = C(10,4)6^6a^{12}x^2.$$

29. Find the middle term in the expansion $(\boldsymbol{1 + 2x + x^2})^{\boldsymbol{7}}$. Level-2(Understanding)

Solution:

$(1 + 2x + x^2)^7 = \{(x+1)^2\}^7 = (x^2+1)^{14}$.

No of terms in this expansion is 15.

8*th term is the middle term of this expansion*.

$t_8 = t_{7+1} = C(14,7)x^7 = \frac{14!}{7!(14-7)!}x^7 = \frac{14\times13\times12\times11\times10\times9\times8\times7!}{7\times6\times5\times4\times3\times2\times1\times7!}x^7 = 3232x^7$.

30. State Binomial theorem for positive integer index. Level-1(Remembering)

Solution:

If x& y are real numbers, then for all $n \in N$,

$(x+y)^n = C(n,0)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + \cdots\cdots + C(n,n)y^n$

i.e. $(x+y)^n = \sum_{r=0}^{n} C(n,r)x^{n-r}y^r,\ 0 \le r \le n$.

Here $C(n,0), C(n,1), C(n,2), \cdots\cdots C(n,n)$are called Binomial coefficients.

Extra Questions (2 Marks)

31. Resolve into partial fractions $\frac{2x+1}{(x-2)(x-3)}$ Level-2(Understanding)

Solution: Let $\frac{2x+1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$

$\Rightarrow 2x + 1 = A(x-3) + B(x-2)$ ------ (1)

Put $x = 2$ *in both sides of equation*(1), *we get* $5 = A(-1) \Rightarrow A =- 5$

Put $x = 3$ *in both sides of equation*(1), *we get* $7 = B(1) \Rightarrow B = 7$

So the required partial fraction

$\frac{2x+1}{(x-2)(x-3)} = \frac{-5}{x-2} + \frac{7}{x-3}$

32. Resolve into partial fractions $\frac{x}{(x+1)(x+3)}$ Level-2(Understanding)

Solution: Let $\frac{x}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} = \frac{A(x+3)+B(x+1)}{(x+1)(x+3)}$

$\Rightarrow x = A(x+3) + B(x+1) ------ (1)$

Put $x = -1$ $in\ both\ sides\ of\ equation(1), we\ get\ \ -1 = A(2) \Rightarrow A = -\frac{1}{2}$

Put $x = -3$ $in\ both\ sides\ of\ equation(1), we\ get\ \ -3 = B(-2) \Rightarrow B = \frac{3}{2}$

So the required partial fraction

$\frac{x}{(x+1)(x+3)} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{3}{2}}{x+3} = -\frac{1}{2(x+1)} + \frac{3}{2(x+3)}$

33. Resolve into partial fractions $\frac{1}{x^2-1}$ Level-2(Understanding)

Solution: Let $\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

$\Rightarrow 1 = A(x+1) + B(x-1) ------ (1)$

Put $x = 1$ $in\ both\ sides\ of\ equation(1), we\ get\ \ 1 = A(2) \Rightarrow A = \frac{1}{2}$

Put $x = -1$ $in\ both\ sides\ of\ equation(1), we\ get\ \ 1 = B(-2) \Rightarrow B = -\frac{1}{2}$

So the required partial fraction

$\frac{1}{x^2-1} = \frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x+1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$

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5 Marks Questions & Solutions ***Taxonomy Level***

1. **If $1, \omega, \omega^2$ are cube roots of unity prove that $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5 = 32$**

Level-3(Applying)

Solution:

L. H. S.

$(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$

$= (1+\omega^2-\omega)^5 + (1+\omega-\omega^2)^5$

$= (-\omega-\omega)^5 + (-\omega^2-\omega^2)^5$ $(\because$ since $1+\omega+\omega^2 = 0 \therefore 1+\omega^2 = -\omega$ and $1+\omega = -\omega^2)$

$= (-2\omega)^5 + (-2\omega^2)^5$

$= -32\omega^5 - 32\omega^{10}$

$= -32\omega^3 \cdot \omega^2 - 32\omega^9 \cdot \omega$

$= -32(\omega^3)\omega^2 - 32(\omega^3)^3 . \omega$

$= -32(1)\omega^2 - 32(1)^3 \cdot \omega$ $(\because$ since $\omega^3 = 1)$

$= -32\omega^2 - 32\omega$

$= -32(\omega^2 + \omega)$

$= -32(-1)$ ($\therefore$ since $1 + \omega + \omega^2 = 0 \therefore \omega^2 + \omega = -1$)

$= 32 = $ (R. H. S)(Proved)

2. **If $x + \frac{1}{x} = 2\cos\theta$, then prove that $x^n + \frac{1}{x^n} = 2\cos n\theta$ and $x^n - \frac{1}{x^n} = 2i\sin(n\theta)$**

Level-3(Applying)

Solution:

Given , $x + \frac{1}{x} = 2\cos\theta$

$$=> \frac{x^2 + 1}{x} = 2\cos\theta$$

$$=> x^2 + 1 = 2x\cos\theta$$

$$=> x^2 - 2x\cos\theta + 1 = 0$$

$$=> x = \frac{-(-2\cos\theta) \pm \sqrt{(-2\cos\theta)^2 - 4(1)(1)}}{2(1)}$$

$$=> x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$$=> x = \frac{2\cos\theta \pm \sqrt{-4(-\cos^2\theta + 1)}}{2}$$

$$=> x = \frac{2\cos\theta \pm \sqrt{-4\sin^2\theta}}{2}$$

$$=> x = \frac{2\cos\theta \pm 2i\sin\theta}{2}$$

$$=> x = \cos\theta \pm i\sin\theta$$

take, $x = \cos\theta + i\sin\theta$

$=> x^n = \cos n\theta + i\sin n\theta$(by De Moivre theorem)

$$=> \frac{1}{x^n} = \frac{1}{\cos n\theta + i\sin n\theta} = (\cos n\theta + i\sin n\theta)^{-1}$$

$=> \frac{1}{x^n} = \cos(-1)(n\theta) + i\sin(-1)(n\theta)$ (by De Moivre's theorem)

$$=> \frac{1}{x^n} = \cos(-n\theta) + i\sin(-n\theta)$$

$$=> \frac{1}{x^n} = \cos(n\theta) - i\sin(n\theta)$$

$$\therefore x^n + \frac{1}{x^n} = \cos n\theta + i\sin n\theta + \cos(n\theta) - i\sin(n\theta) = 2\cos n\theta$$

$$x^n - \frac{1}{x^n} = (\cos n\theta + i\sin n\theta) - (\cos(n\theta) - i\sin(n\theta))$$

$$x^n - \frac{1}{x^n} = \cos n\theta + i\sin n\theta - \cos n\theta + i\sin(n\theta) = 2i\sin n\theta$$

3. **Resolve into pratial fractions**: $\frac{x^2+x-2}{x(x+3)(x-2)}$ **level-2(Understanding)**

Solution:

Let $\frac{x^2+x-2}{x(x+3)(x-2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$

$=> \frac{x^2+x-2}{x(x+3)(x-2)} = \frac{A(x+3)(x-2) + Bx(x-2) + Cx(x+3)}{x(x+3)(x-2)}$

$=> x^2 + x - 2 = A(x+3)(x-2) + Bx(x-2) + Cx(x+3)$

put $x = 0$ $=> -2 = A(0+3)(0-2)$

$=> -2 = A(3)(-2)$

$=> -2 = -6A$

$=> A = \frac{-2}{-6} = \frac{1}{3}$

Put $x = 2 => 2^2 + 2 - 2 = C(2)(2+3)$

$=> 4 + 2 - 2 = C(2)(5)$

$=> 4 = 10C$

$=> C = \frac{4}{10} = \frac{2}{5}$

Put $x = -3 => (-3)^2 - 3 - 2 = B(-3)(-3-2)$

$=> 9 - 3 - 2 = B(-3)(-5)$

$=> 4 = 15B$

$=> B = \frac{4}{15}$

$\therefore \frac{x^2+x-2}{x(x+3)(x-2)} = \frac{\frac{1}{3}}{x} + \frac{\frac{4}{15}}{x+3} + \frac{\frac{2}{5}}{x-2}$

$=> \frac{x^2+x-2}{x(x+3)(x-2)} = \frac{1}{3x} + \frac{4}{15(x+3)} + \frac{2}{5(x-2)}$

4. **Resolve into pratial fractions**: $\frac{1+2x}{(x+2)^2(x-1)^2}$ **level-2(Understanding)**

Solution:

Let $\frac{1+2x}{(x+2)^2(x-1)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$

$$=> \frac{1+2x}{(x+2)^2(x-1)^2} = \frac{A(x+2)(x-1)^2 + B(x-1)^2 + C(x+2)^2(x-1) + D(x+2)^2}{(x+2)^2(x-1)^2}$$

$$=> 1 + 2x = A(x+2)(x-1)^2 + B(x-1)^2 + C(x+2)^2(x-1) + D(x+2)^2$$

Substiute $x - 1 = 0$ i.e. $x = 1$

$$=> 3 = D(3)^2$$

$$=> 3 = D(9)$$

$$=> D = \frac{3}{9} = \frac{1}{3}$$

Again substitute $x + 2 = 0$ i.e. $x =- 2$

$$=> 1 - 4 = B(-3)^2$$

$$=>- 3 = B(3)^2$$

$$=> B = \frac{-3}{9} = \frac{-1}{3}$$

Equating the coefficient of x^3 and x^2 on both sides, we get

$$=> A + C = 0$$

$$=> A =- C$$

And $B + 3C + D = 0$

$$=> \frac{-1}{3} + 3C + \frac{1}{3} = 0$$

$$=> 3C = 0$$

$$=> C = 0$$

$$=> A =- C = 0$$

So our required partial fraction is

$$\frac{1+2x}{(x+2)^2(x-1)^2} = \frac{0}{x+2} + \frac{\frac{-1}{3}}{(x+2)^2} + \frac{0}{x-1} + \frac{\frac{1}{3}}{(x-1)^2}$$

$$\therefore \frac{1+2x}{(x+2)^2(x-1)^2} = \frac{-1}{3(x+2)^2} + \frac{1}{3(x-1)^2}$$

5. Resolve into pratial fractions: $\frac{x^2+6}{(x^2+1)(x^2+4)}$ **Level-2(Understanding)**

Solution:

Let $\frac{x^2+6}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)}$

$$=> \frac{x^2+6}{(x^2+1)(x^2+4)} = \frac{(Ax+B)(x^2+4) + (Cx+D)(x^2+1)}{(x^2+4)(x^2+1)}$$

$$=> x^2 + 6 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1).$$

Equating the coefficient of x^3, x^2, x and the constant terms on both sides, we get

Cofeeicient of x^3 on both side $=> 0 = A + C ------ Eq(1)$

Cofeeicient of x^2 on both side $=> 1 = B + D ------ Eq(2)$

Cofeeicient of x on both side $=> 0 = 4A + C ------ Eq(3)$

Constant term on both side $=> 6 = 4B + D ------ Eq(4)$

From Eq(1) and Eq(3)

We have $A = 0$ and $C = 0$

From Eq(2) and Eq(4)

We have $B = \frac{5}{3}$ and $D = \frac{-2}{3}$

So our required partial fraction is

$$\frac{x^2 + 6}{(x^2 + 1)(x^2 + 4)} = \frac{0x + \frac{5}{3}}{(x^2 + 1)} + \frac{0x + \frac{-2}{3}}{(x^2 + 4)}$$

$$\therefore \frac{x^2 + 6}{(x^2 + 1)(x^2 + 4)} = \frac{5}{3(x^2 + 1)} + \frac{-2}{3(x^2 + 4)}$$

6. **If $P(n, 4) = 2P(5, 3)$, then find the value of n.** **Level-3(Applying)**

Solution:

Given that $P(n, 4) = 2P(5,3)$,

$$=> \frac{n!}{(n-4)!} = 2\frac{5!}{(5-3)!}$$

$$=> \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 2\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

$=> n(n-1)(n-2)(n-3) = 120$

$=> n(n-3)(n-1)(n-2) = 120$

$=> (n^2 - 3n)(n^2 - 3n + 2) = 120$

Substitute $n^2 - 3n = m$, we get

$=> m(m + 2) = 120$

$=> m^2 + 2m - 120 = 0$

$=> m^2 + 12m - 10m - 120 = 0$

$=> m(m + 12) - 10(m + 12) = 0$

$=> (m + 12)(m - 10) = 0$

$=> (m + 12) = 0$ or $(m - 10) = 0$

$=> m = -12$ or $m = 10$

again substitutre the value of m, we get

$=> n^2 - 3n =- 12$ or $n^2 - 3n = 10$

$=> n^2 - 3n + 12 = 0$ or $n^2 - 3n - 10 = 0$

$$=> n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(12)}}{2(1)} \quad \text{or } n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$=> n = \frac{3 \pm \sqrt{9-48}}{2} \quad \text{or } n = \frac{3 \pm \sqrt{9+40}}{2}$$

$$=> n = \frac{3 \pm \sqrt{-39}}{2} \quad \text{or } n = \frac{3 \pm \sqrt{49}}{2}$$

$$=> n = \frac{3 \pm \sqrt{39}i}{2} \quad \text{or } n = \frac{3 \pm 7}{2}$$

$$=> n = \frac{3 + \sqrt{39}i}{2} \text{ and } \frac{3 - \sqrt{39}i}{2} \quad \text{or } n = \frac{3+7}{2} \text{ and } \frac{3-7}{2}$$

$$=> n = \frac{3 + \sqrt{39}i}{2} \text{ and } \frac{3 - \sqrt{39}i}{2} \quad \text{or } n = 5 \text{ and } -2$$

$=> n = 5$ (*negelcting the* $-$ *ve value and the imaginary value*).

7. If $1 \le r \le n$, then prove that $C(n, r) + C(n, r+1) = C(n+1, r+1)$. **Level-3(Applying)**

Proof:

<u>L. H. S.</u>

$$C(n,r) + C(n,r+1) \equiv \frac{n!}{r!\,(n-r)!} \pm \frac{n!}{(r+1)!\,(n-(r+1))!}$$

$$= \frac{n!}{r!\,(n-r) \times (n-r-1)!} + \frac{n!}{(r+1) \times r!\,(n-r-1)!}$$

$$= \frac{n!}{r! \times (n-r-1)!}\left[\frac{1}{n-r} + \frac{1}{(r+1)}\right]$$

$$= \frac{n!}{r! \times (n-r-1)!}\left[\frac{r+1+n-r}{(n-r)(r+1)}\right]$$

$$= \frac{n!}{r! \times (n-r-1)!}\left[\frac{n+1}{(n-r)(r+1)}\right]$$

$$= \frac{n!\,(n+1)}{r! \times (n-r-1)! \times (n-r) \times (r+1)}$$

$$= \frac{(n+1)!}{(n-r)! \times (r+1)!}$$

$$= \frac{(n+1)!}{(r+1)! \times ((n+1)-(r+1))!}$$

$= C(n+1, r+1) = R.H.S. (Proved)$

8. **Find the term independent of x in the expansion $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$.** **Level-2(Understanding)**

Solution:

Let $t_{(r+1)}$th term is independent of x.

But $t_{(r+1)} = (-1)^r C(9,r)\left(\frac{3}{2}x^2\right)^{9-r}\left(\frac{1}{3x}\right)^r$

$$= (-1)^r C(9,r)\left(\frac{3}{2}\right)^{9-r}(x^2)^{9-r}\left(\frac{1}{3}\right)^r\left(\frac{1}{x}\right)^r$$

$$= (-1)^r C(9,r)\frac{3^{9-r}}{2^{9-r}}x^{18-2r}\frac{1}{3^r}\frac{1^r}{x^r}$$

$$= (-1)^r C(9,r)\frac{3^{9-2r}}{2^{9-r}}x^{18-3r}$$

Since this term is independent of x,

$=> 18 - 3r = 0$

$=> 18 = 3r$

$=> r = 6.$

$\therefore t_{(r+1)} = (-1)^r C(9,r)\frac{3^{9-2r}}{2^{9-r}}x^{18-3r}$

$t_{(6+1)} = (-1)^6 C(9,6)\frac{3^{9-2\times6}}{2^{9-6}}x^{18-3\times6}$

$t_7 = \frac{9!}{6!(9-6)!}\frac{3^{9-12}}{2^3}x^0$

$t_7 = \frac{9\times8\times7\times6!}{6!3!}\frac{3^{-3}}{2^3} = \frac{7}{18}$

Hence the 7th term is independent of x and the term is $\frac{7}{18}$.

9. **Find the Square root of $1 + 4\sqrt{3}i$.** **Level-2(Understanding)**

Solution:

Let $\sqrt{1+4\sqrt{3}i} = x + iy$

Squaring both side , we get

$\left(\sqrt{1+4\sqrt{3}i}\right)^2 = (x+iy)^2$

$=> 1 + 4\sqrt{3}i = x^2 + (iy)^2 + 2\times x\times iy$

$=> 1 + 4\sqrt{3}i = x^2 - y^2 + 2\times x\times iy$

Equating the real part and imaginary part on both sides, we get

$x^2 - y^2 = 1$ ------ Eq(1)

$2xy = 4\sqrt{3}$ ------ Eq(2)

$=> (x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$=> (x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$=> (x^2 + y^2)^2 = (1)^2 + \left(4\sqrt{3}\right)^2$

$=> (x^2 + y^2)^2 = 1 + 48$

$=> (x^2 + y^2)^2 = 49$

$=> x^2 + y^2 = 7$ ------- Eq(3)

Solving Eq(1) and Eq(3) , we get

$x = \pm 2$ and $y = \pm \sqrt{3}$

Hence square root of $1 + 4\sqrt{3}i$ i.e.

$\therefore \sqrt{1 + 4\sqrt{3}i} = \pm \left(2 + \sqrt{3}i\right)$.

10. Find the coefficient of $\frac{1}{y^{10}}$ in the expansion $\left(y^3 + \frac{a^7}{y^5}\right)^{10}$. **Level-2(Understanding)**

Solution.

Let $(r + 1)$th tern contains $\frac{1}{y^{10}}$

$$=> t_{(r+1)} = C(10, r)(y^3)^{10-r}\left(\frac{a^7}{y^5}\right)^r$$

$$= C(10, r)y^{30-3r}\ \frac{a^{7r}}{y^{5r}}$$

$$= C(10, r)\frac{a^{7r}}{y^{5r-(30-3r)}}$$

$$= C(10, r)\frac{a^{7r}}{y^{5r-30+3r}}$$

$$= C(10, r)\frac{a^{7r}}{y^{8r-30}}$$

But $y^{8r-30} = y^{10}$

$=> 8r - 30 = 10$

$=> 8r = 30 + 10 = 40$

$$=> r = \frac{40}{8} = 5$$

So,

$$=> t_{(r+1)} = C(10, r)\frac{a^{7r}}{y^{8r-30}}$$

$$=> t_{(5+1)} = C(10,5)\frac{a^{7\times5}}{y^{8\times5-30}}$$

$$=> t_6 = C(10,5)\frac{a^{35}}{y^{10}} = \frac{10!}{5!\,(10-5)!}\frac{a^{35}}{y^{10}}$$

$$= \frac{10\times9\times8\times7\times6\times5!}{5!\,5!}\frac{a^{35}}{y^{10}}$$

$$= \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1}\times a^{35}\times\frac{1}{y^{10}} = 252\times a^{35}\times\frac{1}{y^{10}}$$

Hence the 6th term contains $\frac{1}{y^{10}}$ whose co efficient is $252a^{35}$.

Extra question (5 Marks)

11. Resolve into Partial fraction : $\frac{x^2+1}{x^2-5x+6}$ **Level-2(Understanding)**

Solution: :$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6}$

$\frac{5x-5}{x^2-5x+6}$=$\frac{5x-5}{x^2-3x-2x+6} = \frac{5x-5}{(x-3)(x-2)}$

Let $\frac{5x-5}{(x-3)(x-2)}$ = $\frac{A}{x-3} + \frac{B}{x-2}$ = $\frac{A(x-2)+B(x-3)}{(x-3)(x-2)}$

$\Rightarrow 5x - 5 = A(x-2) + B(x-3)$

Putting x=2 ,we get B = -5 and putting x = 3 ,we get A = 10

Hence $\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6}$ = $1 + \frac{10}{x-3} - \frac{5}{x-2}$

12. Expand $(1 + 2x + x^2)^3$ by using Binomial Theorem . **Level-2(Understanding)**

Solution: $(1 + 2x + x^2)^3$ = $((1 + x)^2)^3 = (1 + x)^6$

= $C(6,0) + C(6,1)x + C(6,2)x^2 + C(6,3)x^3 + C(6,4)x^4 + C(6,5)x^5 + C(6,6)x^6$

= $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$.

13. Find the term independent of x in the expansion of $\left(\frac{x^2}{3} - \frac{4}{x^2}\right)^6$ **Level-2(Understanding)**

Solution: Term independent of x in $\left(\frac{x^2}{3}-\frac{4}{x^2}\right)^6$

Let the term independent of x be $t_{r+1}=C(n,r)\left(\frac{x^2}{3}\right)^{6-r}\left(\frac{-4}{x^2}\right)^r$

$= C(6,r)\frac{x^{12-2r}}{3^{6-r}}(-1)^r 4^r x^{-2r} = C(6,r)(-1)^r\frac{1}{3^{6-r}}4^r x^{12-2r-2r}$

$= C(6,r)(-1)^r\frac{1}{3^{6-r}}4^r x^{12-4r}$

12- 4r = 0

$\Rightarrow$ 4r = 12

$\Rightarrow$ r = 3

The term is $t_4 = C(6,3)(-1)^3\frac{1}{3^{6-3}}4^3 =- 20\times\frac{64}{27}=\frac{-1280}{27}$

14. If α and β are the two roots of the equation $x^2-2x+4=0$, then show that $\alpha^n+\beta^n = 2^{n+1}\,cos\frac{n\pi}{3}$. Level-3(Applying)

Solution:

The roots of the equation $x^2-2x+4=0$ are given by , $x=\frac{2\pm\sqrt{4-16}}{2}=\frac{2\pm2\sqrt{3}\mathrm{i}}{2}=1\pm\sqrt{3}\mathrm{i}$

Let $\alpha=1+\sqrt{3}\mathrm{i}$ and $\beta=1-\sqrt{3}\mathrm{i}$

Now, changing α and β to modulus argument form, we put

1=r cos θ and $\sqrt{3}$ =rsin θ

$r^2(\cos^2\theta+sin^2\theta)=1+3=4 => r=2$

Now, $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3} => \theta=\frac{\pi}{3}$

$\alpha=2\left(cos\frac{\pi}{3}+\mathrm{i}\,sin\frac{\pi}{3}\right)$ and $\beta=2\left(cos\frac{\pi}{3}-\mathrm{i}\,sin\frac{\pi}{3}\right)$

Now, $\alpha^n+\beta^n = 2^n\left[\left(cos\frac{n\pi}{3}+\mathrm{i}\,sin\frac{n\pi}{3}\right)+\left(cos\frac{n\pi}{3}-\mathrm{i}\,sin\frac{n\pi}{3}\right)\right]$

$=2^n 2\cos\frac{n\pi}{3} = 2^{n+1}\cos\frac{n\pi}{3}$. ■